ILLUSTRATIONS OF THE CENTIMETRE-GRAMME-SECOND SYSTEM OF UNITS.

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ILLUSTRATIONS

OF THE

CENTIMETRE-GRAMME-SECOND

(C.G.S.)

SYSTEM OF UNITS,

BASED ON THE RECOMMENDATIONS OF THE COMMITTEE APPOINTED BY THE BRITISH ASSOCIATION "FOR THE SELECTION AND NOMENCLATURE OF DYNAMICAL AND ELECTRICAL UNITS."

BY

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PREFACE.

THE quantitative study of physics, and especially of the relations between different branches of physics, is every day receiving more attention.

To facilitate this study, by exemplifying the use of a system of units fitted for placing such relations in the clearest light, is the main object of the present treatise.

A complete account is given of the theory of units *ab initio.* The Centimetre-Gramme-Second (or C.G.S.) system is then explained; and the remainder of the work is occupied with illustrations of its application to various branches of physics. As a means to this end, the most important experimental data relating to each subject are concisely presented on *one uniform scale*—a luxury hitherto unknown to the scientific calculator.

I am indebted to several friends for assistance in special departments—but especially to Professor Clerk Maxwell and Professor G. C. Foster, who revised the entire manuscript of the work in its original form.

Great pains have been taken to make the work correct as a book of reference. Readers who may discover any errors will greatly oblige me by pointing them out.

J. D. EVERETT.

BELFAST,
June 22nd, 1875.

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TABLES FOR REDUCING OTHER MEASURES TO C.G.S. MEASURES.

The abbreviation *cm.*	is used for	*centimetre* or *centimetres,*
gm.	"	*gramme* or *grammes,*
sec.	"	*second* or *seconds,*
sq.	"	*square,*
cub.	"	*cubic.*

Length.

1 inch	=	2·5400	cm.
1 foot............	=	30·4797	"
1 yard	=	91·4392	"
1 mile	=	160933	"
1 nautical mile....	=	185230	"

More exactly, according to Captain Clarke's comparisons of standards of length (printed in 1866), the metre is equal to 1·09362311 yard, or to 39·370432 inches, the standard metre being taken as correct at 0° C., and the standard yard as correct at $16\frac{2}{3}$° C. Hence the inch is 2·5399772 centimetres.

Area.

1 square inch	=	6·4516	sq. cm.
1 square foot......	=	929·01	"
1 square yard	=	8361·13	"
1 square mile......	=	$2{\cdot}59\times10^{10}$	"

Volume.

1 cubic inch	=	16·387	cub. cm.
1 cubic foot..	=	28316	"
1 cubic yard	=	764535	"
1 pint............	=	567·63	"
1 gallon..........	=	4541	"

Mass.

1 grain............	= ·0647990 gm.
1 ounce avoirdupois	= 28·3495 ,,
1 pound ,,	=453·59 ,,
1 ton..............	$=1{\cdot}01605\times10^6$,,

More exactly, according to the comparisons made by Professor W. H. Miller in 1844 of the "kilogramme des Archives," the standard of French weights, with two English pounds of platinum, and additional weights, also of platinum, the kilogramme is 15432·34874 grains, of which the new standard pound contains 7000. Hence the kilogramme is 2·2046212 pounds, and the pound is 453·59265 grammes.

Velocity.

1 foot per second	=34·4797 cm. per sec.
1 statute mile per hour	=44·704 ,,
1 nautical mile per hour....	=51·453 ,,
1 kilometre per hour	=27·777 ,,

Density.

Pure water at temperature of maximum density	1·000013 gm. per cub. cm.
1 pound per cubic foot	=·016019 ,,

Force (assuming $g=981$).

Weight of 1 grain	= 63·57 dynes, nearly.
,, 1 ounce avoirdupois........	$=2{\cdot}78\times10^4$,,
,, 1 pound avoirdupois........	$=4{\cdot}45\times10^5$,,
,, 1 cwt.	$=4{\cdot}98\times10^7$,,
,, 1 ton..........	$=9{\cdot}97\times10^8$,,
,, 1 gramme	= 981 ,,
,, 1 kilogramme ..	$=9{\cdot}81\times10^5$,,
,, 1 tonne........	$=9{\cdot}81\times10^8$,,

Work (assuming $g=981$).

1 foot-pound	$=1{\cdot}356\times10^7$ ergs, nearly.
1 foot-grain..............	$=1{\cdot}937\times10^3$,,
1 foot-ton	$=3{\cdot}04\times10^{10}$,,
1 milligram-millimetre	$=9{\cdot}81\times10^{-2}$,,
1 gramme-centimetre	$=9{\cdot}81\times10^2$,,
1 kilogrammetre..........	$=9{\cdot}81\times10^7$,,
1 tonne-metre............	$=9{\cdot}81\times10^{10}$,,
Work in a second by one theoretical "horse"	$=7{\cdot}46\times10^9$,,

Pressure (assuming $g=981$).

1 pound per square foot....	$= 479$ dynes per sq. cm., nearly.		
1 pound per square inch ..	$=6 \cdot 9 \times 10^4$	,,	,,
1 kilogramme per square metre	$= 98 \cdot 1$	,,	,,
1 kilogramme per square decimetre	$=9 \cdot 81 \times 10^3$	,,	,,
1 kilogramme per square centimetre	$=9 \cdot 81 \times 10^5$	,,	,,
1 kilogramme per square millimetre	$=9 \cdot 81 \times 10^7$	,,	,,
Pressure of 760 millimetres of mercury at 0° C. ..	$=1 \cdot 014 \times 10^6$	,,	,,

Heat.

1 gramme-degree Centigrade	$=4 \cdot 2 \times 10^7$ ergs $=$ 42 million ergs.
1 pound-degree ,,	$=1 \cdot 91 \times 10^{10}$ ergs.
1 ,, Fahr.	$=1 \cdot 06 \times 10^{10}$ ergs.

ILLUSTRATIONS

OF THE

C.G.S. SYSTEM OF UNITS.

CHAPTER I.

GENERAL THEORY OF UNITS.

Units, and Derived Units.

1. THE *numerical value* of a concrete quantity is its ratio to a selected magnitude of the same kind, called *the unit.*

Thus, if L denote a definite length, and l the unit length, $\frac{L}{l}$ is a true physical ratio, and is called the numerical value of L.

The numerical value of a concrete quantity varies directly as the concrete quantity itself, and inversely as the unit in terms of which it is expressed.

2. A unit of one kind of quantity is sometimes defined by reference to a unit of another kind of quantity. For example, the unit of *area* is commonly defined to be the area of the square described upon the unit of length; and the unit of *volume* is commonly defined as the volume of the cube constructed on the unit of length. The units of area and volume thus defined are called *derived units,* and are more convenient for calculation than independent units would be. For example, when the above definition of the unit of area is employed, we can assert that [the numerical value of] the area of any rectangle is equal to the product of [the numerical values of] its length and breadth; whereas, if any other unit of area were employed, we should have to introduce a third factor which would be constant for all rectangles.

3. Still more frequently, a unit of one kind of quantity is defined by reference to two or more units of other kinds. For example, the unit of *velocity* is commonly defined to be that velocity with which the unit length would be described in the unit time. When we specify a velocity as so many *miles per hour*, or so many *feet per second*, we in effect employ as the unit of velocity a *mile per hour* in the former case, and a *foot per second* in the latter. These are *derived units* of velocity.

Again, the unit *acceleration* is commonly defined to be that acceleration with which a unit of velocity would be gained in a unit of time. The unit of acceleration is thus derived directly from the units of velocity and time, and therefore indirectly from the units of length and time.

4. In these and all other cases, the practical advantage of employing derived units is, that we thus avoid the introduction of additional factors, which would involve needless labour in calculating and difficulty in remembering*.

5. The correlative term to *derived* is *fundamental.* Thus, when the units of area, volume, velocity, and acceleration are defined as above, the units of length and time are called the fundamental units.

Dimensions.

6. Let us now examine the laws according to which derived units vary when the fundamental units are changed.

Let V denote a concrete velocity such that a concrete length L is described in a concrete time T; and let v, l, t denote respectively the unit velocity, the unit length, and the unit time.

Then the numerical value of V is to be equal to the numerical value of L divided by the numerical value of T. But these numerical values are $\frac{V}{v}, \frac{L}{l}, \frac{T}{t}$;

* An example of such needless factors may be found in the rules commonly given in English books for finding the mass of a body when its volume and material are given. "Multiply the volume in cubic feet by the specific gravity and by 62·4, and the product will be the mass in pounds;" or "multiply the volume in cubic inches by the specific gravity and by 253, and the product will be the mass in grains." The factors 62·4 and 253 here employed would be avoided—that is, would be replaced by unity, if the unit volume of water were made the unit of mass.

hence we must have

$$\frac{V}{v}=\frac{L}{l}\frac{t}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

This equation shows that, when the units are changed (a change which does not affect V, L, and T), v must vary directly as l and inversely as t; that is to say, *the unit of velocity varies directly as the unit of length, and inversely as the unit of time.*

Equation (1) also shows that *the numerical value* $\frac{V}{v}$ *of a given velocity* V *varies inversely as the unit of length, and directly as the unit of time.*

7. Again, let A denote a concrete acceleration such that the velocity V is gained in the time T′, and let a denote the unit of acceleration. Then, since the numerical value of the acceleration A is the numerical value of the velocity V divided by the numerical value of the time T′, we have

$$\frac{A}{a}=\frac{V}{v}\frac{t}{T'}.$$

But by equation (1) we may write $\frac{L}{l}\frac{t}{T}$ for $\frac{V}{v}$. We thus obtain

$$\frac{A}{a}=\frac{L}{l}\frac{t}{T}\frac{t}{T'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

This equation shows that when the units a, l, t are changed (a change which will not affect A, L, T, or T′), a must vary directly as l, and inversely in the duplicate ratio of t; and the numerical value $\frac{A}{a}$ will vary inversely as l, and directly in the duplicate ratio of t. In other words, *the unit of acceleration varies directly as the unit of length, and inversely as the square of the unit of time*; and *the numerical value of a given acceleration varies inversely as the unit of length, and directly as the square of the unit of time.*

It will be observed that these have been deduced as direct consequences from the fact that [the numerical value of] an acceleration is equal to [the numerical value of] a length, divided by [the numerical value of] a time, and then again by [the numerical value of] a time.

The relations here pointed out are usually expressed by saying that *the dimensions of acceleration* are $\frac{\text{length}}{(\text{time})^2}$, or that *the dimensions of the unit of acceleration* are $\frac{\text{unit of length}}{(\text{unit of time})^2}$.

8. We have treated these two cases very fully, by way of laying a firm foundation for much that is to follow. We shall hereafter use an abridged form of reasoning, such as the following:—

$$\text{velocity} = \frac{\text{length}}{\text{time}};$$

$$\text{acceleration} = \frac{\text{velocity}}{\text{time}} = \frac{\text{length}}{(\text{time})^2}.$$

Such equations as these may be called *dimensional equations.* Their full interpretation is obvious from what precedes. In all such equations, constant numerical factors can be discarded, as not affecting dimensions.

9. As an example of the application of equation (2) we shall compare the unit acceleration based on the foot and second with the unit acceleration based on the yard and minute.

Let l denote a foot, L a yard, t a second, T a minute, T' a minute. Then a will denote the unit acceleration based on the foot and second, and A will denote the unit acceleration based on the yard and minute. Equation (2) becomes

$$\frac{A}{a} = \frac{3}{1} \times \left(\frac{1}{60}\right)^2 = \frac{1}{1200}; \quad \ldots \ldots \quad (3)$$

that is to say, an acceleration in which a yard per minute of velocity is gained per minute, is $\frac{1}{1200}$ of an acceleration in which a foot per second is gained per second. Or, to use a very expressive notation, which is now becoming common,

$$1 \frac{\text{yard}}{(\text{minute})^2} = \frac{1}{1200} \frac{\text{foot}}{(\text{second})^2}.$$

Meaning of "per."

10. The word *per,* which we have several times employed in the present chapter, denotes division of the quantity named before it by the quantity named after it. Thus, to compute

velocity in feet per second, we must divide a number of feet by a number of seconds *.

If velocity is continuously varying, let x be the number of feet described since a given epoch, and t the number of seconds elapsed, then $\frac{dx}{dt}$ is what is meant by the number of feet *per* second. The word should never be employed in the specification of quantities except when the quantity named before it varies directly as the quantity named after it, at least for small variations—as, in the above instance, the distance described is ultimately proportional to the time of describing it.

Dimensions of Mechanical and Geometrical Quantities.

11. In the following list of dimensions, we employ the letters L, M, T as abbreviations for the words *Length, Mass, Time.* The symbol of equality is used to denote sameness of dimensions.

$$\text{Area} = L^2, \quad \text{Volume} = L^3, \quad \text{Velocity} = \frac{L}{T},$$

$$\text{Acceleration} = \frac{L}{T^2}, \quad \text{Momentum} = \frac{ML}{T}.$$

Density $= \frac{M}{L^3}$, density being defined as mass per unit volume.

Force $= \frac{ML}{T^2}$, since a force is measured by the momentum which it generates per unit of time, and is therefore the quotient of momentum by time—or since a force is measured by the product of a mass by the acceleration generated in this mass.

Work $= \frac{ML^2}{T^2}$, being the product of force and distance.

Kinetic energy $= \frac{ML^2}{T^2}$, being half the product of mass by square of velocity. The constant factor $\frac{1}{2}$ can be omitted, as not affecting dimensions.

Moment of couple $= \frac{ML^2}{T^2}$, being the product of a force by a length.

* It is not correct to speak of interest at the rate of *Five Pounds per cent.* It should be simply *Five per cent.* A rate of five pounds in every hundred pounds is not different from a rate of five shillings in every hundred shillings.

The dimensions of *angle* *, when measured by $\frac{\text{arc}}{\text{radius}}$, are zero. The same angle will be denoted by the same number, whatever be the unit of length employed. In fact we have $\frac{\text{arc}}{\text{radius}} = \frac{L}{L} = L^0$.

The work done by a couple in turning a body through any angle, is the product of the couple by the angle. The identity of dimensions between *work* and *couple* is thus verified.

$$\text{Angular velocity} \quad = \frac{1}{T}.$$

$$\text{Angular acceleration} = \frac{1}{T^2}.$$

$$\text{Moment of inertia} \quad = ML^2.$$

Angular momentum = moment of momentum $= \frac{ML^2}{T}$, being the product of moment of inertia by angular velocity, or the product of momentum by length.

Intensity of pressure, or *intensity of stress* generally, being force per unit of area, is of dimensions $\frac{\text{force}}{\text{area}}$; that is, $\frac{M}{LT^2}$.

Intensity of force of *attraction at a point,* often called simply *force at a point,* being force per unit of attracted mass, is of dimensions $\frac{\text{force}}{\text{mass}}$ or $\frac{L}{T^2}$. It is numerically equal to the acceleration which it generates, and has accordingly the dimensions of acceleration.

The *absolute force of a centre* of attraction, better called the *strength of a centre,* may be defined as the intensity of force at unit distance. If the law of attraction be that of inverse squares, the strength will be the product of the intensity of force at any distance by the square of this distance, and its dimensions will be $\frac{L^3}{T^2}$.

* It is proposed to give the name *radian* to the angle whose arc is equal to radius. Thus, instead of "an angle whose value in circular measure is θ," we shall be able to say "an angle of θ radians."

Curvature (of a curve) $=\frac{1}{L}$, being the angle turned by the tangent per unit distance travelled along the curve.

Tortuosity $=\frac{1}{L}$, being the angle turned by the osculating plane per unit distance travelled along the curve.

The *solid angle* or *aperture* of a conical surface of any form is measured by the area cut off by the cone from a sphere whose centre is at the vertex of the cone, divided by the square of the radius of the sphere. Its dimensions are therefore zero; or a solid angle is a numerical quantity independent of the fundamental units.

The *specific curvature* of a surface at a given point (Gauss's measure of curvature) is the solid angle described by a line drawn from a fixed point parallel to the normal at a point which travels on the surface round the given point, and close to it, divided by the very small area thus enclosed. Its dimensions are therefore $\frac{1}{L^2}$.

The *mean curvature* of a surface at a given point, in the theory of Capillarity, is the arithmetical mean of the curvatures of any two normal sections normal to each other. Its dimensions are therefore $\frac{1}{L}$.

CHAPTER II.

CHOICE OF THREE FUNDAMENTAL UNITS.

12. NEARLY all the quantities with which physical science deals can be expressed in terms of three fundamental units; and the quantities commonly selected to serve as the fundamental units are

a definite length,
a definite mass,
a definite interval of time.

This particular selection is a matter of convenience rather

than of necessity; for any three independent units are theoretically sufficient. For example, we might employ as the fundamental units

a definite mass,
a definite amount of energy,
a definite density.

13. The following are the most important considerations which ought to guide the selection of fundamental units:—

(1) They should be quantities admitting of very accurate comparison with other quantities of the same kind.

(2) Such comparison should be possible at all times. Hence the standards must be permanent—that is, not liable to alter their magnitude with lapse of time.

(3) Such comparison should be possible at all places. Hence the standards must not be of such a nature as to change their magnitude when carried from place to place.

(4) The comparison should be easy and direct.

Besides these experimental requirements, it is also desirable that the fundamental units be so chosen that the definition of the various derived units shall be easy, and their dimensions simple.

14. There is probably no kind of magnitude which so completely fulfils the four conditions above stated as a standard of *mass*, consisting of a piece of gold, platinum, or some other substance not liable to be affected by atmospheric influences. The comparison of such a standard with other bodies of approximately equal mass is effected by weighing, which is, of all the operations of the laboratory, the most exact. Very accurate copies of the standard can thus be secured; and these can be carried from place to place with little risk of injury.

The third of the requirements above specified forbids the selection of a *force* as one of the fundamental units. Forces at the same place can be very accurately measured by comparison with weights; but as gravity varies from place to place, the force of gravity upon a piece of metal, or other standard weight, changes its magnitude in travelling from one place to another. A spring-balance, it is true, gives a direct measure of force; but its indications are too rough for purposes of accuracy.

15. *Length* is an element which can be very accurately

measured and copied. But every measuring-instrument is liable to change its length with temperature. It is therefore necessary, in defining a length by reference to a concrete material standard, such as a bar of metal, to state the temperature at which the standard is correct. The temperature now usually selected for this purpose is that of a mixture of ice and water (0° C.), observation having shown that the temperature of such a mixture is constant.

The length of the standard should not exceed the length of a convenient measuring-instrument; for, in comparing the standard with a copy, the shifting of the measuring-instrument used in the comparison introduces additional risk of error.

In *end-standards,* the standard length is that of the bar as a whole, and the ends are touched by the instrument every time that a comparison is made. This process is liable to wear away the ends and make the standard false. In *line-standards,* the standard length is the distance between two scratches, and the comparison is made by optical means. The scratches are usually at the bottom of holes sunk halfway through the bar.

16. *Time* is also an element which can be measured with extreme precision. The direct instruments of measurement are clocks and chronometers; but these are checked by astronomical observations, and especially by transits of stars. The time between two successive transits of a star is (very approximately) the time of the earth's rotation on its axis; and it is upon the uniformity of this rotation that the preservation of our standards of time depends.

Necessity for a Common Scale.

17. The existence of quantitative correlations between the various forms of energy, imposes upon men of science the duty of bringing all kinds of physical quantity to one common scale of comparison. Several such measures (called *absolute* measures) have been published in recent years; and a comparison of them brings very prominently into notice the great diversity at present existing in the selection of particular units of length, mass, and time.

Sometimes the units employed have been the foot, the grain,

and the second; sometimes the millimetre, milligramme, and second; sometimes the centimetre, gramme, and second; sometimes the centimetre, gramme, and minute; sometimes the metre, tonne, and second; sometimes the metre, gramme, and second; while sometimes a mixture of units has been employed; the area of a plate, for example, being expressed in square metres, and its thickness in millimetres.

A diversity of scales may be tolerable, though undesirable, in the specification of such simple matters as length, area, volume, and mass when occurring singly; for the reduction of these from one scale to another is generally understood. But when the quantities specified involve a reference to more than one of the fundamental units, and especially when their dimensions in terms of these units are not obvious, but require careful working out, there is great increase of difficulty and of liability to mistake.

A general agreement as to the particular units of length, mass, and time which shall be employed—if not in all scientific work, at least in all work involving complicated references to units—is urgently needed; and almost any one of the selections above instanced would be better than the present option.

18. We shall adopt the recommendation of the Units Committee of the British Association (see Appendix), that all specifications be referred to the *Centimetre*, the *Gramme*, and the *Second*. The system of units derived from these as the fundamental units is called the *C.G.S. system*; and the units of the system are called the *C.G.S. units*.

The reason for selecting the centimetre and gramme rather than the metre and gramme, is that, since a gramme of water has a volume of approximately 1 cubic centimetre, the former selection makes the density of water unity; whereas the latter selection would make it a million, and the density of a substance would be a million times its specific gravity, instead of being identical with its specific gravity as in the C.G.S. system.

Even those who may have a preference for some other units will nevertheless admit the advantage of having a variety of results, from various branches of physics, reduced from their original multiplicity and presented in one common scale.

19. The adoption of one common scale for all quantities in-

volves the frequent use of very large and very small numbers. Such numbers are most conveniently written by expressing them as the product of two factors, one of which is a power of 10; and it is usually advantageous to effect the resolution in such a way that the exponent of the power of 10 shall be the characteristic of the logarithm of the number. Thus 3240000000 will be written $3{\cdot}24 \times 10^9$, and ·00000324 will be written $3{\cdot}24 \times 10^{-6}$.

CHAPTER III.

MECHANICAL UNITS.

Value of g.

20. ACCELERATION is defined as the rate of increase of velocity per unit of time. The C.G.S. unit of acceleration is the acceleration of a body whose velocity increases in every second by the C.G.S. unit of velocity—namely, by a centimetre per second. The apparent acceleration of a body falling freely under the action of gravity in vacuo is denoted by g. The value of g in C.G.S. units at any part of the earth's surface is approximately given by the following formula,

$$g = 980{\cdot}6056 - 2{\cdot}5028 \cos 2\lambda - {\cdot}000003h,$$

λ denoting the latitude, and h the height of the station (in centimetres) above sea-level.

The constants in this formula have been deduced from numerous pendulum experiments in different localities, the length l of the seconds' pendulum being connected with the value of g by the formula $g = \pi^2 l$.

Dividing the above equation by π^2 we have, for the length of the seconds' pendulum, in centimetres,

$$l = 99{\cdot}3562 - {\cdot}2536 \cos 2\lambda - {\cdot}0000003h.$$

At sea-level these formulæ give the following values for the places specified:—

	Latitude.	Value of g.	Value of l.
Equator	0° 0′	978·10	99·103
Latitude 45°	45 0	980·61	99·356
Munich	48 9	980·88	99·384
Paris	48 50	980·94	99·390
Greenwich	51 29	981·17	99·413
Göttingen	51 32	981·17	99·414
Berlin	52 30	981·25	99·422
Dublin	53 21	981·32	99·429
Manchester	53 29	981·34	99·430
Belfast	54 36	981·43	99·440
Edinburgh	55 57	981·54	99·451
Aberdeen	57 9	981·64	99·461
Pole	90 0	983·11	99·610

The difference between the greatest and least values (in the case of both g and l) is about $\frac{1}{196}$ of the mean value.

Force.

21. The C.G.S. unit of force is called the *dyne*. It is the force which, acting upon a gramme for a second, generates a velocity of a centimetre per second.

It may otherwise be defined as the force which, acting upon a gramme, produces the C.G.S. unit of acceleration, or as the force which, acting upon any mass for 1 second, produces the C.G.S. unit of momentum.

To show the equivalence of these three definitions, let m denote mass in grammes, v velocity in centimetres per second, t time in seconds, F force in dynes.

Then, by the second law of motion, we have

$$\text{acceleration} = \frac{\text{force}}{\text{mass}};$$

that is, if α denote acceleration in C.G.S. units, $\alpha = \frac{F}{m}$; hence, when α and m are each unity, F will be unity.

Again, by the nature of uniform acceleration, we have $v = at$, v denoting the velocity due to the acceleration a, continuing for time t.

Hence we have $F = ma = \frac{mv}{t}$. Therefore, if $mv = 1$ and $t = 1$, we have $F = 1$.

As a particular case, if $m = 1$, $v = 1$, $t = 1$, we have $F = 1$.

22. The force represented by the *weight of a gramme* varies from place to place. It is the force required to sustain a gramme in vacuo, and would be nil at the earth's centre, where gravity is nil. To compute its amount in dynes at any place where g is known, observe that a mass of 1 gramme falls in vacuo with acceleration g. The force producing this acceleration (namely, the weight of the gramme) must be equal to the product of the mass and acceleration, that is to g.

The weight (when weight means force) of 1 gramme is therefore g dynes; and the weight of m grammes is mg dynes.

23. Force is said to be expressed in *gravitation-measure* when it is expressed as equal to the weight of a given mass. Such specification is inexact unless the value of g is also given. For purposes of accuracy it must always be remembered that the pound, the gramme, &c. are, strictly speaking, units of mass. Such an expression as "a force of 100 tons" must be understood as an abbreviation for "a force equal to the weight [at the locality in question] of 100 tons."

Work and Energy.

24. The C.G.S. unit of work is called the *erg*. It is the amount of work done by a dyne working through a distance of a centimetre.

The C.G.S. unit of energy is also the erg, energy being measured by the amount of work which it represents.

25. To establish a rule for computing the *kinetic energy* (or *energy due to the motion*) of a given mass moving with a given velocity, it is sufficient to consider the case of a body falling in vacuo.

When a body of m grammes falls through a height of h centimetres, the working force is the weight of the body—that is,

gm dynes, which, multiplied by the distance worked through, gives gmh ergs as the work done. But the velocity acquired is such that $v^2 = 2gh$. Hence we have $gmh = \frac{1}{2}mv^2$.

The kinetic energy of a mass of m grammes moving with a velocity of v centimetres per second is therefore $\frac{1}{2}mv^2$ ergs; that is to say, this is the amount of work which would be required to generate the motion of the body, or is the amount of work which the body would do against opposing forces before it would come to rest.

26. Work, like force, is often expressed in *gravitation-measure*. Gravitation units of work, such as the foot-pound and kilogramme-metre, vary with locality, being proportional to the value of g.

One gramme-centimetre is equal to g ergs.

One kilogramme-metre is equal to 100,000 g ergs.

One foot-pound is $\cdot 13825 \times 10^5 \times g$ ergs.

Taking g as 981, this is $1\cdot 356 \times 10^7$ ergs.

26 A. The unit rate of working is 1 erg per second. Watt's "horse-power" is defined as 550 foot-pounds per second. This is $7\cdot 46 \times 10^9$ ergs per second. The "force de cheval" is defined as 75 kilogrammetres per second. This is $7\cdot 36 \times 10^9$ ergs per second. We here assume $g = 981$.

Examples.

1. If a spring balance is graduated so as to show the masses of bodies in pounds or grammes when used at the equator, what will be its error when used at the poles, neglecting effects of temperature?

Ans. Its indications will be too high by about $\frac{1}{196}$ of the total weight.

2. A cannon-ball, of 10,000 grammes, is discharged with a velocity of 45,000 centims. per sec. Find its kinetic energy.

Ans. $\frac{1}{2} \times 10000 \times (45000)^2 = 1\cdot 0125 \times 10^{13}$ ergs.

3. In last question find the mean force exerted upon the ball by the powder, the length of the barrel being 200 centims.

Ans. $5\cdot 0625 \times 10^{10}$ dynes.

4. Given that 42 million ergs are equivalent to 1 gramme-

degree of heat, and that a gramme of lead at 10° C. requires 15·6 gramme-degrees of heat to melt it; find the velocity with which a leaden bullet must strike a target that it may just be melted by the collision, supposing all the mechanical energy of the motion to be converted into heat and to be taken up by the bullet.

We have $\frac{1}{2}v^2 = 15 \cdot 6 \times J$, where $J = 42 \times 10^6$. Hence $v^2 = 1310$ millions; $v = 36 \cdot 2$ thousand centims. per second.

5. With what velocity must a stone be thrown vertically upwards at a place where g is 981 that it may rise to a height of 3000 centims.? and to what height would it ascend if projected vertically with this velocity at the surface of the moon, where g is 150?

Ans. 2426 centims. per second; 19620 centims.

Centrifugal Force.

27. A body moving in a curve must be regarded as continually falling away from a tangent. The acceleration with which it falls away is $\frac{v^2}{r}$, v denoting its velocity and r the radius of curvature. The acceleration of a body in any direction is always due to force urging it in that direction, this force being equal to the product of mass and acceleration. Hence the normal force on a body of m grammes moving in a curve of radius r centimetres, with velocity v centimetres per second, is $\frac{mv^2}{r}$ dynes. This force is directed towards the centre of curvature. The equal and opposite force with which the body reacts is called centrifugal force.

If the body moves uniformly in a circle, the time of revolution being T seconds, we have $v = \frac{2\pi r}{T}$; hence $\frac{v^2}{r} = \left(\frac{2\pi}{T}\right)^2 r$, and the force acting on the body is $mr\left(\frac{2\pi}{T}\right)^2$ dynes.

If n revolutions are made per minute, the value of T is $\frac{60}{n}$, and the force is $mr\left(\frac{n\pi}{30}\right)^2$ dynes.

Examples.

1. A body of m grammes moves uniformly in a circle of radius 80 centims., the time of revolution being $\frac{1}{4}$ of a second. Find the centrifugal force, and compare it with the weight of the body.

Ans. The centrifugal force is $m \times \left(\frac{2\pi}{\frac{1}{4}}\right)^2 \times 80 = m \times 64\,\pi^2 \times 80 = 50540\,m$ dynes.

The weight of the body (at a place where g is 981) is 981 m dynes. Hence the centrifugal force is about 52 times the weight of the body.

2. At a bend in a river, the velocity in a certain part of the surface is 170 centims. per second, and the radius of curvature of the lines of flow is 9100 centims. Find the slope of the surface in a section transverse to the lines of flow.

Ans. Here the centrifugal force for a gramme of the water is $\frac{(170)^2}{9100} = 3{\cdot}176$ dynes. If g be 981 the slope will be $\frac{3{\cdot}176}{981} = \frac{1}{309}$; that is, the surface will slope upwards from the concave side at a gradient of 1 in 309. The general rule applicable to questions of this kind is that the resultant of centrifugal force and gravity must be normal to the surface.

3. An open vessel of liquid is made to rotate rapidly round a vertical axis. Find the number of revolutions that must be made per minute in order to obtain a slope of 30° at a part of the surface distant 10 centims. from the axis, the value of g being 981.

Ans. We must have $\tan 30° = \frac{f}{g}$, where f denotes the intensity of centrifugal force—that is, the centrifugal force per unit mass. We have therefore

$$981 \tan 30° = 10\left(\frac{n\pi}{30}\right)^2, \quad n \text{ denoting the number of revolutions per minute,}$$

$$= \frac{n^2\pi^2}{90}.$$

Hence $n = 71{\cdot}9$.

4. For the intensity of centrifugal force at the equator due to

the earth's rotation, we have $r=$ earth's radius $=6{\cdot}38\times10^6$, $T=86164$ being the number of seconds in a sidereal day;

$$\therefore f=r\left(\frac{2\pi}{T}\right)^2=3{\cdot}39.$$

This is about $\frac{1}{289}$ of the value of g.

If the earth were at rest, the value of g at the equator would be greater than at present by this amount. If the earth were revolving about 17 times as fast as at present, the value of g at the equator would be nil.

CHAPTER IV.

HYDROSTATICS.

28. The following Table of the relative density of water at various temperatures (under atmospheric pressure), the density at 4° C. being taken as unity, is from Rossetti's results deduced from all the best experiments (Ann. Ch. Phys. x. 461; xvii. 370, 1869):—

Temp. Cent.	Relative Density.	Temp. Cent.	Relative Density.	Temp. Cent.	Relative Density.
0°	·999871	13°	·999430	35°	·99418
1	·999928	14	·999299	40	·99235
2	·999969	15	·999160	45	·99037
3	·999991	16	·999002	50	·98820
4	1·000000	17	·998841	55	·98582
5	·999990	18	·998654	60	·98338
6	·999970	19	·998460	65	·98074
7	·999933	20	·998259	70	·97794
8	·999886	22	·997826	75	·97498
9	·999824	24	·997367	80	·97194
10	·999747	26	·996866	85	·96879
11	·999655	28	·996331	90	·96556
12	·999549	30	·995765	100	·95865

29. According to Kupffer's observations, as reduced by Professor W. H. Miller, the absolute density (in grammes per cubic centimetre) at 4° is not 1, but 1·000013. Multiplying the above numbers by this factor, we obtain the following Table of absolute densities:—

Temp.	Density.	Temp.	Density.	Temp.	Density.
0°	·999884	13°	·999443	35°	·99469
1	·999941	14	·999312	40	·99236
2	·999982	15	·999173	45	·99038
3	1·000004	16	·999015	50	·98821
4	1·000013	17	·998854	55	·98583
5	1·000003	18	·998667	60	·98339
6	·999983	19	·998473	65	·98075
7	·999946	20	·998272	70	·97795
8	·999899	22	·997839	75	·97499
9	·999837	24	·997380	80	·97195
10	·999760	26	·996879	85	·96880
11	·999668	28	·996344	90	·96557
12	·999562	30	·995778	100	·95866

30. The volume, at temperature $t°$, of the water which occupies unit volume at 4°, is approximately

$$1+A(t-4)^2-B(t-4)^{2\cdot 6}+C(t-4)^3,$$

where

$$A=8{\cdot}38\times 10^{-6},$$

$$B=3{\cdot}79\times 10^{-7},$$

$$C=2{\cdot}24\times 10^{-8};$$

and the relative density at temperature $t°$ is given by the same formula with the signs of A, B, and C reversed. The rate of expansion at temperature $t°$ is

$$2A(t-4)-2{\cdot}6B(t-4)^{1\cdot 6}+3C(t-4)^2.$$

In determining the signs of the terms with the fractional exponents 2·6 and 1·6, these exponents are to be regarded as odd.

31. Table of Densities (chiefly taken from Rankine's 'Rules and Tables,' pp. 149 & 150):—

Solids.

Brass, cast	7·8 to 8·4	Ice	·92
„ wire	8·54	Basalt	3·00
Bronze	8·4	Brick	2 to 2·17
Copper, cast	8·6	Brickwork	1·8
„ sheet	8·8	Chalk	1·8 to 2·8
„ hammered	8·9	Clay	1·92
Gold	19 to 19·6	Glass, crown	2·5
Iron, cast	6·95 to 7·3	„ flint	3·0
„ wrought	7·6 to 7·8	Quartz (rock-crystal)	2·65
Lead	11·4	Sand (dry)	1·42
Platinum	21 to 22	Fir, spruce	·48 to ·7
Silver	10·5	Oak, European	·69 to ·99
Steel	7·8 to 7·9	Lignum vitæ	·65 to 1·33
Tin	7·3 to 7·5	Sulphur, octahedral	2·05
Zinc	6·8 to 7·2	„ prismatic	1·98

Liquids at 0° C.

Sea-water, ordinary	1·026	Naphtha	·848
Alcohol, pure	·791	Oil, linseed	·940
„ proof spirit	·916	„ olive	·915
Ether	·716	„ whale	·923
Mercury	13·596	„ of turpentine	·870

32. If a body weighs m grammes in vacuo and m' grammes in water of density unity, the volume of the body is $m-m'$ cubic centims.; for the mass of the water displaced is $m-m'$ grammes, and each gramme of this water occupies a cubic centimetre.

Examples.

1. A glass cylinder, l centims. long, weighs m grammes in vacuo and m' grammes in water of unit density. Find its radius.

Solution. Its section is πr^2, and is also $\frac{m-m'}{l}$; hence

$$r^2 = \frac{m-m'}{\pi l}.$$

2. Find the capacity at 0° C. of a bulb which holds m grammes of mercury at that temperature.

Solution. The specific gravity of mercury at 0° being 13·596 as compared with water at the temperature of maximum density, it follows that the mass of 1 cubic centim. of mercury is $13·596 \times 1·000013 = 13·59618$, say 13·596. Hence the required capacity is $\frac{m}{13·596}$ cubic centims.

3. Find the total pressure on a surface whose area is A square centims. when its centre of gravity is immersed to a depth of h centims. in water of unit density, atmospheric pressure being neglected.

Ans. Ah grammes weight; that is, gAh dynes.

4. If mercury of specific gravity 13·596 is substituted for water in the preceding question, find the pressure.

Ans. 13·596 Ah grammes weight; that is, 13·596 gAh dynes.

5. If h be 76, and A be unity in example 4, the answer becomes 1033·3 grammes weight, or 1033·3g dynes.

For Paris, where g is 980·94, this is $1{\cdot}0136 \times 10^6$ dynes.

Barometric Pressure.

33. The C.G.S. unit of pressure-intensity (that is, of pressure per unit area) is the pressure of a dyne per square centim.

At the depth of h centims. in a uniform liquid whose density is d [grammes per cubic centim.], the pressure due to the weight of the liquid is ghd dynes per square centim.

The pressure-intensity due to the weight of a column of mercury at 0° C., 76 centims. high, is found by putting h=76, d=13·596, and is 1033·3g. It is therefore different at different localities. At Paris, where g is 980·94, it is $1{\cdot}0136 \times 10^6$; that is, rather more than a megadyne * per square centim. To exert a pressure of exactly one megadyne per square centim., the height of the column at Paris must be 74·98 centims.

At Greenwich, where g is 981·17, the pressure due to 76 centims. of mercury at 0° C. is $1{\cdot}0138 \times 10^6$; and the height which would give a pressure of 10^6, is 74·964 centims., or 29·514 inches.

Convenience of calculation would be promoted by adopting the pressure of a megadyne per square centim., or 10^6 C.G.S. units of pressure-intensity, as the standard atmosphere.

The standard now commonly adopted (whether 76 centims. or 30 inches) denotes different pressures at different places, the pressure denoted by it being proportional to the value of g.

We shall adopt the megadyne per square centim. as our standard atmosphere in the present work.

* The prefix *mega* denotes multiplication by a million. A megadyne is a force of a million dynes.

Examples.

1. What must be the height of a column of water of unit density to exert a pressure of a megadyne per square centim. at a place where g is 981?

Ans. $\frac{1000000}{981}=1019{\cdot}4$ centims. This is 33·445 feet.

2. What is the pressure due to an inch of mercury at 0° C. at a place where g is 981? (An inch is 2·54 centims.)

Ans. $981\times 2{\cdot}54\times 13{\cdot}596=33878$ dynes per square centim.

3. What is the pressure due to a centim. of mercury at 0° C. at the same locality?

Ans. $981\times 13{\cdot}596=13338$.

4. What is the pressure due to a kilometre of sea-water of density 1·027, g being 981?

Ans. $981\times 10^5\times 1{\cdot}027 = 1{\cdot}0075\times 10^8$ dynes per square centim., or $1{\cdot}0075\times 10^2$ megadynes per square centim.; that is, about 100 atmospheres.

5. What is the pressure due to a mile of the same water?

Ans. $1{\cdot}6214\times 10^8$ C.G.S. units, or 162·14 atmospheres [of a megadyne per square centim.].

Density of Air.

34. Regnault found that at Paris, under the pressure of a column of mercury at 0°, of the height of 76 centims., the density of perfectly dry air was ·0012932 gramme per cubic centim. The pressure corresponding to this height of the barometer at Paris is $1{\cdot}0136\times 10^6$ dynes per square centim. Hence, by Boyle's law, we can compute the density of dry air at 0° C. at any given pressure.

At a pressure of a megadyne (10^6 dynes) per square centim. the density will be $\frac{{\cdot}0012932}{1{\cdot}0136}={\cdot}0012759$.

The density of dry air at 0° C. at any pressure p (dynes per square centim.) is

$$p\times 1{\cdot}2759\times 10^{-9} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Example.

Find the density of dry air at 0° C., at Edinburgh, under

the presssure of a column of mercury at 0° C., of the height of 76 centims.

Here we have $p = 981{\cdot}54 \times 76 \times 13{\cdot}596 = 1{\cdot}0142 \times 10^6$.

Ans. Required density $= 1{\cdot}2940 \times 10^{-3} = {\cdot}0012940$ gramme per cubic centim.

35. *Absolute Densities of Gases, in grammes per cubic centim., at* 0° C., *and a pressure of* 10^6 *dynes per square centim.*

	Mass of a cubic centim. in grammes.		Volume of a gramme in cubic centims.
Air, dry	·0012759	......	783·8
Oxygen	·0014107	......	708·9
Nitrogen............	·0012393	......	806·9
Hydrogen	·00008837	......	11316·0
Carbonic acid........	·0019509	......	512·6
„ oxide	·0012179	......	821·1
Marsh-gas	·0007173	......	1394·1
Chlorine............	·0030909	......	323·5
Protoxide of nitrogen .	·0019433	......	514·6
Binoxide „	·0013254	......	754·5
Sulphurous acid......	·0026990	......	370·5
Cyanogen	·0022990	......	435·0
Olefiant gas	·0012529	......	798·1
Ammonia	·0007594	......	1316·8

The numbers in the second column are the reciprocals of those in the first.

The numbers in the first column are identical with the specific gravities referred to water as unity.

Assuming that the densities of gases at constant pressure and temperature are directly as their atomic weights, we have for any gas at zero $pv\mu = 1{\cdot}1316 \times 10^{10} m$; v denoting its volume in cubic centims., m its mass in grammes, p its pressure in dynes per square centim., and μ its atomic weight referred to that of hydrogen as unity.

Height of Homogeneous Atmosphere.

36. We have seen that the intensity of pressure at depth h, in a fluid of uniform density d, is ghd when the pressure at the upper surface of the fluid is zero.

The atmosphere is not a fluid of uniform density; but it is often convenient to have a name to denote a height H such that $p = g\text{HD}$, where p denotes the pressure and D the density of the air at a given point.

It may be defined as the height of a column of uniform fluid having the same density as the air at the point, which would exert a pressure equal to that existing at the point.

If the pressure be equal to that exerted by a column of mercury of density 13·596 and height h, we have

$$p = gh \times 13{\cdot}596;$$

$$\therefore \; HD = h \times 13{\cdot}596, \quad H = \frac{h \times 13{\cdot}596}{D}.$$

If it were possible for the whole body of air above the point to be reduced by vertical compression to the density which the air has at the point, the height from the point up to the summit of this compressed atmosphere would be equal to H, subject to a small correction for the variation of gravity with height.

H is called the *height of the homogeneous atmosphere* at the point considered. *Pressure-height* would be a better name.

The general formula for it is

$$H = \frac{p}{gD}; \quad \ldots \ldots \ldots \quad (5)$$

and this formula will be applicable to any other gas as well as dry air, if we make D denote the density of the gas (in grammes per cubic centim.) at pressure p.

If, instead of p being given directly in dynes per square centim., we have given the height h of a column of liquid of density d which would exert an equal pressure, the formula reduces to

$$H = \frac{hd}{D}. \quad \ldots \ldots \ldots \quad (6)$$

37. The value of $\frac{p}{D}$ in formula (5) depends only on the nature of the gas and on the temperature; hence, for a given gas at a given temperature, H varies inversely as g.

For dry air at zero we have, by formula (4),

$$\frac{p}{D} = \frac{10^9}{1{\cdot}2759} = 7{\cdot}8376 \times 10^8;$$

$$\therefore \; H = \frac{7{\cdot}8376 \times 10^8}{g}.$$

At Paris, where g is 980·94, we find

$$H = 7\cdot990 \times 10^5.$$

At Greenwich, where g is 981·17,

$$H = 7\cdot988 \times 10^5.$$

Examples.

1. Find the height of the homogeneous atmosphere at Paris for dry air at 10° C., and also at 100° C.

Ans. For given density p varies as $1 + \cdot00366\, t$, t denoting the temperature on the Centigrade scale. Hence we have, at 10° C.,

$$H = 1\cdot0366 \times 7\cdot99 \times 10^5 = 8\cdot2825 \times 10^5;$$

and at 100° C.,

$$H = 1\cdot366 \times 7\cdot99 \times 10^5 = 1\cdot0914 \times 10^6.$$

2. Find the height of the homogeneous atmosphere for hydrogen at 0°, at a place where g is 981.

Here we have

$$H = \frac{p}{gd} = \frac{10^6}{981 \times 8837 \times 10^{-5}} = 1\cdot1535 \times 10^7.$$

Diminution of Density with increase of Height in the Atmosphere.

38. Neglecting the variation of gravity with height, the variation of H as we ascend in the atmosphere would depend only on variation of temperature. In an atmosphere of uniform temperature H will be the same at all heights. In such an atmosphere, an ascent of 1 centim. will involve a diminution of the pressure (and therefore of the density) by $\frac{1}{H}$ of itself, since the layer of air which has been traversed is $\frac{1}{H}$ of the whole mass of superincumbent air. The density therefore diminishes by the same fraction of itself for every centim. that we ascend; in other words, the density and pressure diminish in geometrical progression as the height increases in arithmetical progression.

Denote height above a fixed level by x, and pressure by p. Then, in the notation of the differential calculus, we have

$$\frac{dx}{\mathrm{H}} = -\frac{dp}{p};$$

and if p_1, p_2 are the pressures at the heights x_1, x_2, we deduce

$$x_2 - x_1 = \mathrm{H} \log_e \frac{p_1}{p_2} = \mathrm{H} \times 2{\cdot}3026 \log_{10} \frac{p_1}{p_2}. \quad . \quad . \quad (7)$$

In the barometric determination of heights it is usual to compute H by assuming a temperature which is the arithmetical mean of the temperatures at the two heights.

For the latitude of Greenwich formula (7) becomes

$$x_2 - x_1 = (1 + {\cdot}00366\, t)\; 7{\cdot}988 \times 10^5 \times 2{\cdot}3026 \log \frac{p_1}{p_2}$$

$$= (1 + {\cdot}00366\, t)\; 1{,}839{,}300 \log \frac{p_1}{p_2}, \quad . \quad . \quad . \quad (8)$$

t denoting the mean temperature, and the logarithms being common logarithms.

To find the height at which the density would be halved, variations of temperature being neglected, we must put 2 for $\frac{p_1}{p_2}$ in these formulæ. The required height will be $\mathrm{H} \log_e 2$, or, in the latitude of Greenwich, for temperature 0° C., will be

$$1{\cdot}8393 \times 10^6 \times {\cdot}30103 = 553700.$$

The value of $\log_e 2$, or $2{\cdot}3026 \log_{10} 2$, is

$$2{\cdot}3026 \times {\cdot}30103 = {\cdot}69315.$$

Hence for an atmosphere of any gas at uniform temperature, the height at which the density would be halved is the height of the homogeneous atmosphere for that gas, multiplied by ·69315. The gas is assumed to obey Boyle's law.

Examples.

1. Show that if the pressure of the gas at the lower station and the value of g be given, the height at which the density will be halved varies inversely as the density.

2. At what height, in an atmosphere of hydrogen at 0° C., would the density be halved, g being 981 ?

Ans. $7{\cdot}9954 \times 10^6$.

39. *Pressure of Aqueous Vapour at various temperatures, in dynes per square centim.*

−20°	1236	50°	$1{\cdot}226 \times 10^5$
−15	1866	60	1·985 „
−10	2790	80	4·729 „
−5	4150	100	$1{\cdot}014 \times 10^6$
0	6133	120	1·988 „
5	8710	140	3·626 „
10	12220	160	6·210 „
15	16930	180	$1{\cdot}006 \times 10^7$
20	23190	200	1·560
25	31400		
30	42050		
40	73200		

The density of saturated steam, at any temperature t, is approximately

$$\frac{{\cdot}622 \times {\cdot}0012759}{1 + {\cdot}00366\,t} \times \frac{p}{10^6},$$

p denoting the pressure as given in the above Table.

40. *Pressure of Vapour of various Liquids, in dynes per square centim.*

	Alcohol.	Ether.	Sulphide of Carbon.	Chloroform.
−20°	4455	$9{\cdot}19 \times 10^4$	$6{\cdot}31 \times 10^4$	
−10	8630	$1{\cdot}53 \times 10^5$	$1{\cdot}058 \times 10^5$	
0	16940	2·46 „	1·706 „	
10	32310	3·826 „	2·648 „	
20	59310	5·772 „	3·975 „	$2{\cdot}141 \times 10^5$
30	$1{\cdot}048 \times 10^5$	8·468 „	5·799 „	3·301 „
40	1·783 „	$1{\cdot}210 \times 10^6$	8·240 „	4·927 „
50	2·932 „	1·687 „	$1{\cdot}144 \times 10^6$	7·14 „
60	4·671 „	2·301 „	1·554 „	$1{\cdot}007 \times 10^6$
80	$1{\cdot}084 \times 10^6$	4·031 „	2·711 „	1·878 „
100	2·265 „	6·608 „	4·435 „	3·24 „
120	4·31 „	$1{\cdot}029 \times 10^7$	6·87 „	5·24 „

41. The phenomena of capillarity, soap-bubbles, &c. can be

reduced to quantitative expression by assuming a tendency in the surface of every liquid to contract. The following Table exhibits the intensity of this contractile force for various liquids at the temperature of 20° C. The contractile force diminishes as the temperature increases.

Superficial Tensions at 20° C., in dynes per linear centim., deduced from Quincke's results.

	Density.	Tension of surface separating the liquid from		
		Air.	Water.	Mercury.
Water	0·9982	81	0	418
Mercury................	13·5432	540	418	0
Bisulphide of carbon	1·2687	32·1	41·75	372·5
Chloroform	1·4878	30·6	29·5	399
Alcohol	·7906	25·5	....	399
Olive-oil................	·9136	36·9	20·56	335
Turpentine.............	·8867	29·7	11·55	250·5
Petroleum	·7977	31·7	27·8	284
Hydrochloric acid........	1·1	70·1	....	377
Solution of hyposulphite of soda	1·1248	77·5	....	442·5

CHAPTER V.

COEFFICIENTS OF ELASTICITY.

42. Every coefficient of elasticity is a stress divided by a strain—the stress being expressed in units of force per unit of area, while the strain is a mere numerical quantity. We shall always suppose stresses to be expressed in dynes per square centim.

43. The *coefficient of volume-elasticity* of a liquid is the quotient of the pressure per unit area by the compression produced, the compression being denoted by the fraction which the difference of volumes is of the original volume. The fol-

lowing values are reduced from those given in Jamin, 'Cours de Physique,' tom. i. pp. 168 & 169 :—

	Temp. Cent.	Coefficient of Volume-elasticity.	Compression for one Atmosphere (megadyne per square centim.).
Mercury	$0°0$	$3·436 \times 10^{11}$	$2·91 \times 10^{-6}$
Water............	0·0	$2·02 \times 10^{10}$	$4·96 \times 10^{-5}$
"	1·5	1·97 "	5·08 "
"	4·1	2·03 "	4·92 "
"	10·8	2·11 "	4·73 "
"	13·4	2·13 "	4·70 "
"	18·0	2·20 "	4·55 "
"	25·0	2·22 "	4·50 "
"	34·5	2·24 "	4·47 "
"	43·0	2·29 "	4·36 "
"	53·0	2·30 "	4·35 "
Ether	0·0	$9·2 \times 10^{9}$	$1·09 \times 10^{-4}$
	0·0	7·8 "	1·29 "
Alcohol	14·0	7·2 "	1·38 "
	7·3	$1·22 \times 10^{10}$	$8·17 \times 10^{-5}$
	13·1	1·12 "	8·91 "
Sea-water	17·5	2·33 "	4·30 "

44. The following are reduced from the results obtained by Amaury and Descamps, 'Comptes Rendus,' tom. lxviii. p. 1564 (1869), and are probably more accurate than the foregoing, especially in the case of mercury :—

		Coefficient of Elasticity.	Compression for one megadyne per square centim.
Distilled water	$15°$	$2·22 \times 10^{10}$	$4·51 \times 10^{-5}$
Alcohol	0	1·21 "	8·24 "
"	15	1·11 "	8·99 "
Ether	0	$9·30 \times 10^{9}$	$1·08 \times 10^{-4}$
"	14	7·92 "	1·26 "
Bisulphide of carbon	14	$1·60 \times 10^{10}$	$6·26 \times 10^{-5}$
Mercury	15	$5·42 \times 10^{11}$	$1·84 \times 10^{-6}$

Coefficients of Elasticity for Solids.

45. When the stress is a simple longitudinal stress, and the strain considered is the extension or compression in the direction

of the stress, the quotient $\frac{\text{stress}}{\text{strain}}$ is called "Young's Modulus of Elasticity," or briefly "Young's Modulus."

When the stress is a shearing stress, and the strain is the shear produced, the planes of the stress being the same as the planes of the shear, the quotient $\frac{\text{stress}}{\text{strain}}$ is called the "simple rigidity" for these planes.

When the stress consists in hydrostatic pressure, and the strain is the compression produced, the quotient $\frac{\text{stress}}{\text{strain}}$ is called the "elasticity of volume," or the "coefficient of volume-elasticity," just as in the case of a liquid.

46. The values in the following Table are reduced from those given in my own papers to the Royal Society (see Phil. Trans. 1867, p. 369), by employing the value of g at the place of observation, namely 981·4.

	Young's Modulus.	Simple Rigidity.	Elasticity of Volume.	Density.
Glass, flint	$6{\cdot}03 \times 10^{11}$	$2{\cdot}40 \times 10^{11}$	$4{\cdot}15 \times 10^{11}$	2·942
Another specimen.	5·74 ,,	2·35 ,,	3·47 ,,	2·935
Brass, drawn	$1{\cdot}075 \times 10^{12}$	3·66 ,,	....	8·471
Steel............	2·139 ,,	8·19 ,,	$1{\cdot}841 \times 10^{12}$	7·849
Iron, wrought....	1·963 ,,	7·69 ,,	1·456 ,,	7·677
,, cast	1·349 ,,	5·32 ,,	$9{\cdot}64 \times 10^{11}$	7·235
Copper..........	1·234 ,,	4·47 ,,	$1{\cdot}684 \times 10^{12}$	8·843

47. The following are reduced from Sir W. Thomson's results (Proc. Roy. Soc. May 1865), the value of g being 981·4:—

	Simple Rigidity.			
Brass, three specimens..	4·03	3·48	3·44	$\times 10^{11}$
Copper, two specimens..	4·40	4·40		$\times 10^{11}$

Other specimens of copper in abnormal states gave results ranging from $3{\cdot}86 \times 10^{11}$ to $4{\cdot}64 \times 10^{11}$.

48. The following are reduced from Wertheim's results (Ann. de Chim. ser. 3. tom. xxiii.), g being taken as 981:—

Different specimens of glass (crystal).

Young's modulus....	3·41 to 4·34,	mean 3·96	$\times 10^{11}$
Simple rigidity......	1·26 to 1·66,	,, 1·48	$\times 10^{11}$
Volume-elasticity ..	3·50 to 4·39,	,, 3·89	$\times 10^{11}$

Different specimens of brass.

Young's modulus....	9·48 to 10·44, mean	9·86	$\times 10^{11}$
Simple rigidity......	3·53 to 3·90, „	3·67	
Volume-elasticity ..	10·02 to 10·85, „	10·43	

49. Savart's experiments on the torsion of brass wire (Ann. de Chim. 1829) lead to the value $3{\cdot}61 \times 10^{11}$ for simple rigidity.

Kupffer's values of Young's modulus, for nine different specimens of brass, range from $7{\cdot}96 \times 10^{11}$ to $11{\cdot}4 \times 10^{11}$, the value generally increasing with the density.

For a specimen, of density 8·4465, the value was $10{\cdot}58 \times 10^{11}$.

For a specimen, of density 8·4930, the value was $11{\cdot}2 \times 10^{11}$.

The values of Young's modulus found by the same experimenter for steel, range from $20{\cdot}2 \times 10^{11}$ to $21{\cdot}4 \times 10^{11}$.

50. The following are reduced from Rankine's 'Rules and Tables,' pp. 195 & 196, the mean value being adopted where different values are given :—

	Tenacity.	Young's Modulus.
Steel bars......	$7{\cdot}93 \times 10^{9}$	$2{\cdot}45 \times 10^{12}$
Iron wire......	5·86 „	1·745 „
Copper wire....	4·14 „	1·172 „
Brass wire	3·38 „	$9{\cdot}81 \times 10^{11}$
Lead, sheet	$2{\cdot}28 \times 10^{8}$	$5{\cdot}0 \times 10^{10}$
Tin, cast	3·17 „	......
Zinc	5·17 „	......
Ash	$1{\cdot}172 \times 10^{9}$	$1{\cdot}10 \times 10^{11}$
Spruce	$8{\cdot}55 \times 10^{8}$	1·10 „
Oak	$1{\cdot}026 \times 10^{9}$	1·02 „
Glass..........	$6{\cdot}48 \times 10^{8}$	$5{\cdot}52 \times 10^{11}$
Brick and cement	$2{\cdot}0 \times 10^{7}$	......

The tenacity of a substance may be defined as the greatest longitudinal stress that it can bear without tearing asunder. The quotient of the tenacity by Young's modulus will therefore be the greatest longitudinal extension that the substance can bear.

CHAPTER VI.

ASTRONOMY.

51. *Dimensions of the Earth, in centims.*

Polar axis	$1{\cdot}27122 \times 10^9$.
Greater equatorial axis	$1{\cdot}27566$ „
Less „	$1{\cdot}27533$ „
Mean equatorial diameter	$1{\cdot}2755$ „
Mean of the three axes	$1{\cdot}27407$ „
Mean radius	$6{\cdot}3703 \times 10^8$.
Maximum quadrant of meridian	$1{\cdot}000134 \times 10^9$.
Minimum „	$1{\cdot}000028$ „
Minute of latitude	185200 − 940 cos 2 lat. of middle of arc.
Volume of earth	$1{\cdot}08287 \times 10^{27}$ cubic centims.
Mass, assuming 5·67 as mean density.	$6{\cdot}14 \times 10^{27}$ grammes.

Day and Year.

Sidereal day	86164	mean solar seconds.
Sidereal year	31,558,150	„
Tropical year	31,556,929	„

Angular velocity of earth's rotation	$\frac{2\pi}{86164}=\frac{1}{13713}$.
Velocity of points on the equator due to earth's rotation	46506 centims. per second.
Velocity of earth in orbit, about	2933000 „
Centrifugal force at equator due to earth's rotation	3·3912 dynes per gramme.

Attraction in Astronomy.

52. The mass of the moon is the product of the earth's mass by ·011364, and is therefore to be taken as $6{\cdot}98 \times 10^{25}$ grammes, the doubtful element being the earth's mean density, which we take as 5·67.

The mean distance of the centres of gravity of the earth and moon is 60·2734 equatorial radii of the earth—that is, $3{\cdot}8439 \times 10^{10}$ centims.

The mean distance of the sun from the earth is about $1{\cdot}473 \times 10^{13}$ centims., or 91·52 million miles, corresponding to a parallax of $8''{\cdot}93$.

The intensity of centrifugal force due to the earth's motion in its orbit (regarded as circular) is $\left(\frac{2\pi}{T}\right)^2 r$, r denoting the mean

distance, and T the length of the sidereal year, expressed in seconds. This is equal to the acceleration due to the sun's attraction at this distance. Putting for r and T their values, $1{\cdot}473 \times 10^{13}$ and $3{\cdot}1558 \times 10^{7}$, we have

$$\left(\frac{2\pi}{T}\right)^2 r = {\cdot}5839.$$

This is about $\frac{1}{1680}$ of the value of g at the earth's surface.

The intensity of the earth's attraction at the mean distance of the moon is about

$$\frac{981}{(60{\cdot}27)^2} \text{ or } {\cdot}2701.$$

This is less than the intensity of the sun's attraction upon the earth and moon, which is ·5839 as just found. Hence the moon's path is always concave towards the sun.

53. The mutual attractive force F between two masses m and m', at distance l, is

$$F = C\frac{mm'}{l^2}$$

where C is a constant. To determine its value, consider the case of a gramme at the earth's surface, attracted by the earth. Then we have

$$F = 981,\ m = 1,\ m' = 6{\cdot}14 \times 10^{27},\ l = 6{\cdot}37 \times 10^{8};$$

whence we find

$$C = \frac{6{\cdot}48}{10^8} = \frac{1}{1{\cdot}543 \times 10^7}.$$

To find the mass m which, at the distance of 1 centim. from an equal mass, would attract it with a force of 1 dyne, we have

$$1 = Cm^2;$$

whence

$$m = \sqrt{\frac{1}{C}} = 3928 \text{ grammes.}$$

54. To find the acceleration α produced at the distance of l centims. by the attraction of a mass of m grammes, we have

$$\alpha = \frac{F}{m'} = C\frac{m}{l^2},$$

where C has the value $6{\cdot}48 \times 10^{-8}$ as above.

To find the dimensions of C we have $C=\frac{r^2 a}{m}$, where the dimensions of a are LT^{-2}.
The dimensions of C are therefore

$$L^2M^{-1}LT^{-2}; \text{ that is, } L^3M^{-1}T^{-2}.$$

If C^{-1} gramme were adopted as the unit of mass, the centimetre and second being still retained as the units of length and time, we should have

$$\text{acceleration} = \frac{\text{mass}}{(\text{distance})^2}.$$

The value of C^{-1} is $1{\cdot}543\times10^7$. The unit of mass here suggested is therefore about 15 tonnes. Its dimensions are L^2LT^{-2}; that is, L^3T^{-2}. When this unit of mass is employed, the strength of a centre of attraction (p. 6, last paragraph) is numerically equal to the mass which may be supposed to be collected there.

The mass of the earth on this system is the product of the acceleration due to gravity at the earth's surface, and the square of the earth's radius. This product is

$$981\times(6{\cdot}37\times10^8)^2=3{\cdot}98\times10^{20},$$

and is independent of determinations of the earth's density.

The new unit of force will be the force which, acting upon the new unit of mass, produces unit acceleration. It will therefore be equal to $1{\cdot}543\times10^7$ dynes; and its dimensions will be

$$MLT^{-2}=L^3T^{-2}LT^{-2}=L^4T^{-4}.$$

It is thus theoretically possible to base a general system of units upon two fundamental units alone; one of the three fundamental units being eliminated by means of the equation

$$\text{mass} = \text{acceleration}\times(\text{distance})^2.$$

Such a system would be eminently convenient in astronomy, but could not be applied with accuracy to ordinary terrestrial purposes, because we can only roughly compare the earth's mass with the masses which we weigh in our balances.

55. If we adopt a new unit of length equal to l centims., and a new unit of time equal to t seconds, while we define the

unit mass as that which produces unit acceleration at unit distance, the unit mass will be equal to

$$l^3 t^{-2} \times 1{\cdot}543 \times 10^7 \text{ grammes.}$$

If we make l the wave-length of the line F in vacuo, say

$$4{\cdot}86 \times 10^{-5},$$

and t the period of vibration of the same ray; that is to say, the wave-length divided by the velocity of light, or

$$4{\cdot}86 \times 10^{-5} \div (3 \times 10^{10}) = 1{\cdot}62 \times 10^{-15},$$

the unit mass will be

$$(4{\cdot}86 \times 10^{-5})^3 \times (1{\cdot}62 \times 10^{-15})^{-2} \times (1{\cdot}543 \times 10^7)$$
$$= 6{\cdot}75 \times 10^{23} \text{ grammes.}$$

The ratio of this quantity to the mass of the earth is $\frac{1}{9100}$, and is independent of the earth's density.

CHAPTER VII.

VELOCITY OF SOUND.

56. The propagation of sound through any medium is due to the elasticity of the medium; and the general formula for the velocity of propagation s is

$$s = \sqrt{\frac{E}{D}},$$

where D denotes the density of the medium, and E the coefficient of elasticity.

57. For air, or any gas, we are to understand by E, the quotient

$$\frac{\text{increment of pressure}}{\text{corresponding compression}};$$

that is to say, if P, P $+p$ be the initial and final pressures, and V, V $-v$ the initial and final volumes, p and v being small in comparison with P and V, we have

$$E = \frac{p}{\frac{v}{V}} = p\frac{V}{v}.$$

If the compression took place at constant temperature, we should have

$$\frac{p}{P} = \frac{v}{V} \text{ and } E = P.$$

But in the progagation of sound, the compression is effected so rapidly that there is not time for any sensible part of the heat of compression to escape, and we have

$$\frac{p}{P} = \gamma\frac{v}{V}, \quad E = \gamma P, \quad s = \sqrt{\frac{\gamma P}{D}},$$

where $\gamma = 1{\cdot}41$ for dry air, oxygen, nitrogen, or hydrogen.

The value of $\frac{P}{D}$ for dry air at t° Cent. (see p. 21) is

$$(1 + {\cdot}00366\, t) \times 7{\cdot}838 \times 10^8.$$

Hence the velocity of sound through dry air is

$$s = 10^4 \sqrt{1{\cdot}41 \times (1 + {\cdot}00366\, t) \times 7{\cdot}838}$$
$$= 33240 \sqrt{1 + {\cdot}00366\, t};$$

or approximately, for atmospheric temperatures,

$$s = 33240 + 60\, t.$$

58. In the case of any liquid, E denotes the coefficient of volume-elasticity *.

For water at $8^\circ{\cdot}1$ C. (the temperature of the Lake of Geneva in Colladon's experiment) we have

$$E = 2{\cdot}08 \times 10^{10}, \quad D = 1 \text{ sensibly};$$
$$\therefore \sqrt{\frac{E}{D}} = \sqrt{E} = 144000.$$

The velocity as determined experimentally by Colladon was 143500.

59. For the propagation of sound along a solid, in the form of a thin rod, wire, or pipe, which is free to expand or contract laterally, E must be taken as denoting Young's modulus of elasticity *. The values of E and D will be different for different

* Strictly speaking, E should be taken as denoting the coefficient of elasticity for sudden applications of stress—so sudden that there is not time for changes of temperature produced by the stress to be sensibly diminished by conduction. This remark applies to both §§ 58 and 59. For the amount of these changes of temperature, see a later section under Heat.

specimens of the same material. Employing the values given in the Table (§ 46), we have

	Values of E.	Values of D.	Values of $\sqrt{\frac{E}{D}}$, or velocity.
Glass, first specimen ..	$6{\cdot}03 \times 10^{11}$	2·942	$4{\cdot}53 \times 10^{5}$
„ second specimen .	5·74 „	2·935	4·42 „
Brass	$1{\cdot}075 \times 10^{12}$	8·471	3·56 „
Steel	2·139 „	7·849	5·22 „
Iron, wrought	1·963 „	7·677	5·06 „
„ cast	1·349 „	7·235	4·32 „
Copper	1·234 „	8·843	3·74 „

60. If the density of a specimen of red pine be ·5, and its modulus of longitudinal elasticity be $1{\cdot}6 \times 10^6$ pounds per square inch at a place where g is 981, compute the velocity of sound in the longitudinal direction.

By the Table at the commencement of the present volume, a pound per square inch (g being 981) is $6{\cdot}9 \times 10^4$ dynes per square centim. Hence we have for the required velocity

$$\sqrt{\frac{E}{D}} = \sqrt{\frac{1{\cdot}6 \times 10^6 \times 6{\cdot}9 \times 10^4}{{\cdot}5}} = 4{\cdot}7 \times 10^5$$

centims. per second.

61. The following numbers, multiplied by 10^5, are the velocities of sound through the principal metals, as determined by Wertheim :—

	At 20° C.	At 100° C.	At 200° C.
Lead	1·23	1·20	
Gold	1·74	1·72	1·73
Silver................	2·61	2·64	2·48
Copper	3·56	3·29	2·95
Platinum	2·69	2·57	2·46
Iron	5·13	5·30	4·72
Iron wire (ordinary)....	4·92	5·10	
Cast steel	4·99	4·92	4·79
Steel wire (English) ..	4·71	5·24	5·00
„	4·88	5·01	

The following velocities in wood are from the observations

of Wertheim and Chevandier, 'Comptes Rendus,' 1846, pp. 667 & 668:—

	Along Fibres.	Radial Direction.	Tangential Direction.
Pine	$3{\cdot}32\times10^5$	$2{\cdot}83\times10^5$	$1{\cdot}59\times10^5$
Beech..........	3·34 „	3·67 „	2·83 „
Witch-elm......	3·92 „	3·41 „	2·39 „
Birch	4·42 „	2·14 „	3·03 „
Fir	4·64 „	2·67 „	1·57 „
Acacia	4·71 „		
Aspen..........	5·08 „		

62. *Musical Strings.*

Let M denote the mass of a string per unit length,
F „ stretching force,
L „ length of the vibrating portion;
then the velocity with which pulses travel along the string is

$$v=\sqrt{\frac{F}{M}},$$

and the number of vibrations made per second is

$$n=\frac{v}{2L}.$$

Example.

For the 4 strings of a violin the values of M in grammes per centimetre of length are

$$\cdot00416,\quad \cdot00669,\quad \cdot0106,\quad \cdot0266.$$

The values of n are

$$660,\quad 440,\quad 293\tfrac{1}{3},\quad 195\tfrac{5}{9};$$

and the common value of L is 33 centims. Hence the values of v or $2Ln$ are

$$43560,\quad 29040,\quad 19360,\quad 12910$$

centims. per second; and the values of F or Mv^2, in dynes, are

$$7{\cdot}89\times10^6,\quad 5{\cdot}64\times10^6,\quad 3{\cdot}97\times10^6,\quad 4{\cdot}43\times10^6.$$

CHAPTER VIII.

LIGHT.

63. All kinds of light have the same velocity in vacuo. According to the most recent experiments by Cornu (see 'Nature,' February 4, 1875) this velocity is $3{\cdot}004\times 10^{10}$ centims. per second. Foucault's determination was $2{\cdot}98\times 10^{10}$.

The velocity of light of given refrangibility in any medium is the quotient of the velocity in vacuo by the absolute index of refraction for light of the given refrangibility in that medium. If then μ denote this index, the velocity will be

$$\frac{3{\cdot}004\times 10^{10}}{\mu}.$$

Light of given refrangibility is light of given wave-frequency. Its wave-length in any medium is the quotient of the velocity in that medium by the wave-frequency. If n denote the wave-frequency (that is to say, the number of waves which traverse a given point in one second), the wave-length will be

$$\frac{3{\cdot}004\times 10^{10}}{n\mu}.$$

64. The following are the wave-lengths adopted by Ångström for the principal Fraunhofer lines in air at 760 millims. pressure (at Upsal) and 16° C.:—

		centims.	
A	....	7·604	$\times 10^{-5}$
B	....	6·867	"
C	....	6·56201	"
Mean of lines D	....	5·89212	"
E	....	5·26913	"
F	....	4·86072	"
G	....	4·30725	"
H_1	....	3·96801	"
H_2	....	3·93300	"

These numbers will be approximately converted into the corresponding wave-lengths in vacuo by multiplying them by 1·00029.

65. The formula established by the experiments of Biot and Arago for the index of refraction of air was

$$\mu - 1 = \frac{{\cdot}0002943}{1+\alpha t}\cdot\frac{h}{760};$$

t denoting the temperature Centigrade, α the coefficient of expansion ·00366, and h the pressure in millims. of mercury at zero. As the pressure of 760 millims. of such mercury at Paris is $1{\cdot}0136 \times 10^6$ dynes per square centim., the general formula applicable to all localities alike will be

$$\mu - 1 = \frac{\cdot 0002943}{1 + \cdot 00366\,t} \cdot \frac{P}{1{\cdot}0136 \times 10^6},$$

where P denotes the pressure in C.G.S. units. This can be reduced to the form

$$\mu - 1 = \frac{\cdot 0002903}{1 + \cdot 00366\,t} \cdot \frac{P}{10^6} \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

66. Adopting $\frac{3{\cdot}004 \times 10^{10}}{1{\cdot}00029}$, that is $3{\cdot}0033 \times 10^{10}$, as the velocity of light in air, and neglecting the difference of velocity between the more and less refrangible rays, we obtain the following quotients of velocity in air by wave-length :—

	Vibrations per second.
A	$3{\cdot}950 \times 10^{14}$
B	$4{\cdot}373$,,
C	$4{\cdot}577$,,
D	$5{\cdot}097$,,
E	$5{\cdot}700$,,
F	$6{\cdot}179$,,
G	$6{\cdot}973$,,
H_1	$7{\cdot}569$,,
H_2	$7{\cdot}636$,,

CHAPTER IX.

HEAT.

67. The unit of heat is usually defined as the quantity of heat required to raise, by one degree, the temperature of unit mass of water, initially at a certain standard temperature. The standard temperature usually employed is 0° C.; but this is liable to the objection that ice may be present in water at this

temperature. Hence 4° C. has been proposed as the standard temperature; and another proposition is to employ as the unit of heat one hundredth part of the heat required to raise the unit mass of water from 0° to 100° C.

68. According to Regnault (Mém. Acad. Sciences, xxi. p. 729) the quantity of heat required to raise a given mass of water from 0° to t° C. is proportional to

$$t + \cdot 00002\, t^2 + \cdot 0000003\, t^3. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

The *mean thermal capacity* of a body *between two stated temperatures* is the quantity of heat required to raise it from the lower of these temperatures to the higher, divided by the difference of the temperatures. The mean thermal capacity of a given mass of water between 0° and t° is therefore proportional to

$$1 + \cdot 00002\, t + \cdot 0000003\, t^2. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

The *thermal capacity* of a body *at a stated temperature* is the limiting value of the mean thermal capacity as the range is indefinitely diminished. Hence the thermal capacity of a given mass of water at t° is proportional to the differential coefficient of (1), that is to

$$1 + \cdot 00004\, t + \cdot 0000009\, t^2. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Hence the thermal capacities at 0° and 4° are as 1 to 1·000174 nearly; and the thermal capacity at 0° is to the mean thermal capacity between 0° and 100° as 1 to 1·005.

69. If we agree to adopt the capacity of unit mass of water at a stated temperature as the unit of capacity, the unit of heat must be defined as n times the quantity of heat required to raise unit mass of water from this initial temperature through $\frac{1}{n}$ of a degree, when n is indefinitely great.

Supposing the standard temperature and the length of the degree of temperature to be fixed, the units both of heat and of thermal capacity vary directly as the unit of mass.

In what follows, we adopt as the unit of heat (except where the contrary is stated) the heat required to raise a gramme of pure water through 1° C. at a temperature intermediate between 0°

and 4°. This specification is sufficiently precise for the statement of any thermal measurements hitherto made.

70. The *thermal capacity of unit mass* of a substance at any temperature is called the *specific heat* of the substance at that temperature.

The following determinations of specific heat by Dulong and Petit agree very well with later determinations by Regnault and other experimenters, except in the case of platinum :—

	Mean Specific Heat between 0° and 100°.	Mean Specific Heat between 0° and 300°.
Iron	·1098	·1218
Copper	·0949	·1013
Zinc	·0927	·1015
Silver	·0557	·0611
Antimony	·0507	·0549
Platinum	·0355	·0355
Glass	·1770	·1990

According to Pouillet's experiments, the mean specific heat of platinum between

0° and	100°	is	·0335
„	300	„	·0343
„	500	„	·0352
„	700	„	·0360
„	1000	„	·0373
„	1200	„	·0382

71. Specific heat is of zero dimensions in length, mass, and time. It is in fact the ratio

$$\frac{\text{increment of heat in the substance}}{\text{increment of heat in water}}$$

for a given increment of temperature, the comparison being between *equal masses* of the substance at the actual temperature and of water at the standard temperature. The numerical value of a given concrete specific heat merely depends upon the standard temperature at which the specific heat of water is called unity.

72. The *thermal capacity of unit volume* of a substance is another important element, we shall denote it by c. Let s denote the specific heat, and d the density of the substance; then c is the thermal capacity of d units of mass, and therefore $c=sd$. The dimensions of c in length, mass, and time are the same as

those of d, namely $\frac{M}{L^3}$. Its numerical value will not be altered by any change in the units of length, mass, and time which leaves the value of the density of water unchanged.

In the C.G.S. system, since the density of water between 0° and 4° is very approximately unity, the thermal capacity of unit volume of a substance is the value of the ratio

$$\frac{\text{increment of heat in the substance}}{\text{increment of heat in water}}$$

for a given increment of temperature, when the comparison is between *equal volumes.*

Conductivity.

73. By the *thermal conductivity* of a substance *at a given temperature* is meant the value of k in the expression

$$Q = kA\frac{v_2 - v_1}{x}t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where Q denotes the quantity of heat that flows, in time t, through a plate of the substance of thickness x, the area of each of the two opposite faces of the plate being A, and the temperatures of these faces being respectively v_1 and v_2, each differing but little from the given temperature. The lines of flow of heat are supposed to be normal to the faces, or, in other words, the isothermal surfaces within the plate are supposed to be parallel to the faces; and the flow of heat is supposed to be *steady,* in other words, no part of the plate is to be gaining or losing heat on the whole.

Briefly, and subject to these understandings, conductivity may be defined as *the quantity of heat that passes, in unit time, through unit area of a plate whose thickness is unity, when its opposite faces differ in temperature by one degree.*

74. *Dimensions of conductivity.* From equation (1) we have

$$k = \frac{Q}{v_2 - v_1} \cdot \frac{x}{At}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The dimensions of the factor $\frac{Q}{v_2 - v_1}$ are simply M, since the unit

of heat varies jointly as the unit of mass and the length of the degree. The dimensions of the factor $\frac{x}{At}$ are $\frac{1}{LT}$; hence the dimensions of k are $\frac{M}{LT}$. This is on the supposition that the unit of heat is the heat required to raise *unit mass* of water one degree. In calculations relating to conductivity it is perhaps more usual to adopt as the unit of heat the heat required to raise *unit volume* of water one degree. The dimensions of $\frac{Q}{v_2-v_1}$ will then be L^3, and the dimensions of k will be $\frac{L^2}{T}$.

These conclusions may be otherwise expressed by saying that the dimensions of conductivity are $\frac{M}{LT}$ when the thermal capacity of unit mass of water is taken as unity, and are $\frac{L^2}{T}$ when the capacity of unit volume of water is taken as unity. In the C.G.S. system the capacities of unit mass and unit volume of water are practically identical.

75. Let c denote the thermal capacity of unit volume of a substance through which heat is being conducted. Then $\frac{k}{c}$ denotes a quantity whose value it is often necessary to discuss in investigations relating to the transmission of heat. We have, from equation (2),

$$\frac{k}{c}=\frac{Q'}{v_2-v_1}\cdot\frac{x}{At},$$

where Q' denotes $\frac{Q}{c}$. Hence $\frac{k}{c}$ would be the numerical value of the conductivity of the substance, if the unit of heat employed were the heat required to raise unit volume of the substance one degree. Professor Clerk Maxwell proposes to call $\frac{k}{c}$ the *thermometric* conductivity, as distinguished from k, which is the *thermal* conductivity. $\frac{k}{c}$ is strictly analogous to the *coefficient of diffusion* of a substance, which is defined as follows:—"If the opposite sides a and b of a stratum of thickness x are maintained

in the states A and B respectively, and if the rate of diffusion is such that the quantity diffused in time t from a to b, if applied to a stratum of thickness y, would bring it into the state A, then the coefficient of diffusion is $\frac{xy}{t}$." If the thing diffused is heat, and the states A and B are temperatures, this is precisely the definition of $\frac{k}{c}$. The dimensions of $\frac{xy}{t}$ are clearly $\frac{L^2}{T}$; and we obtain the same result by dividing the dimensions of k by those of c; for we have

$$\frac{M}{LT} \div \frac{M}{L^3} = \frac{L^2}{T}.$$

Results of Experiments on Conductivity.

76. Principal Forbes's results for the conductivity of iron (Stewart on Heat, p. 261, second edition) are expressed in terms of the foot and minute, the cubic foot of water being the unit of thermal capacity. Hence the value of Forbes's unit of conductivity, when referred to C.G.S., is $\frac{(30{\cdot}48)^2}{60}$, or 15·48; and his results must be multiplied by 15·48 to reduce them to the C.G.S. scale. His observations were made on two square bars; the side of the one being $1\frac{1}{4}$ inch, and of the other an inch. The results when reduced to C.G.S. units are as follows:—

Temp. Cent.	$1\frac{1}{4}$-inch bar.	1-inch bar.
0°	·207	·1536
25	·1912	·1460
50	·1771	·1399
75	·1656	·1339
100	·1567	·1293
125	·1496	·1259
150	·1446	·1231
175	·1399	·1206
200	·1356	·1183
225	·1317	·1160
250	·1279	·1140
275	·1240	·1121

77. Neumann's results (Ann. de. Chim. vol. lxvi. p. 185) must be multiplied by ·000848 to reduce them to our scale. They then become as follows:—

	Conductivity.
Copper	1·108
Brass	·302
Zinc	·307
Iron	·164
German silver	·109
Ice	·0057

In the same paper he gives for the following substances the values of $\frac{k}{sd}$ or $\frac{k}{c}$; that is, the quotient of conductivity by the thermal capacity of unit volume. These require the same reducing factor as the values of k, and when reduced to our scale are as follows :—

	Values of $\frac{k}{c}$.
Oil	·00116
Melted sulphur	·00142
Ice	·0114
Snow	·00356
Frozen mould	·00916
Sandy loam	·0136
Granite (coarse)	·0109
Serpentine	·00594

78. Sir W. Thomson's results, deduced from observations of underground thermometers at three stations at Edinburgh (Trans. R. S. E. 1860, p. 426), are given in terms of the foot and second, the thermal capacity of a cubic foot of water being unity, and must be multiplied by $(30{\cdot}48)^2$ or 929 to reduce them to our scale. The following are the reduced results :—

	k, or Conductivity.	$\frac{k}{c}$.
Trap-rock of Calton Hill	·00415	·00786
Sand of experimental garden	·00262	·00872
Sandstone of Craigleith Quarry	·01068	·02311

My own result for the value of $\frac{k}{c}$ from the Greenwich underground thermometers (Greenwich Observations, 1860) is in terms of the French foot and the year. As a French foot is 32·5 centims., and a year is 31557000 seconds, the reducing factor is $(32{\cdot}5)^2 \div 31557000$; that is, $3{\cdot}347 \times 10^{-5}$. The reduced result is

	$\frac{k}{c}$
Gravel of Greenwich Observatory Hill	·01249

79. Ångström, in Pogg. Ann. vols. cxiv. (1861) and cxviii. (1863), employs as units the centimetre and the minute; hence his results must be divided by 60. These results, as given at p. 429 of his second paper, will then stand as follows:—

	Value of $\frac{k}{c}$.
Copper, first specimen	$1{\cdot}216\ (1 - {\cdot}00214\,t)$
„ second specimen	$1{\cdot}163\ (1 - {\cdot}001519\,t)$
Iron	${\cdot}224\ (1 - {\cdot}002874\,t)$

He adopts for c the values

·84476 for copper; ·88620 for iron,

and thus deduces the following values of k:—

	Conductivity.
Copper, first specimen	$1{\cdot}027\ (1 - {\cdot}00214\,t)$
„ second specimen	${\cdot}983\ (1 - {\cdot}001519\,t)$
Iron	${\cdot}199\ (1 - {\cdot}002874\,t)$

80. In Professor George Forbes's paper on conductivity (Proc. R. S. E., February 1873) the units are the centim. and the minute; hence his results must be divided by 60. Thus reduced, they are:—

Ice, along axis	·00223
Ice, perpendicular to axis	·00213
Black marble	·00177
White marble	·00115
Slate	·00081
Snow	·00072
Cork	·000717
Glass	·0005
Pasteboard	·000453
Carbon	·000405
Roofing-felt	·000335
Fir, parallel to fibre	·0003
Fir, across fibre and along radius	·000088
Boiler-cement	·000162
Paraffin	·00014
Sand, very fine	·000131
Sawdust	·000123
Kamptulikon	·00011
Vulcanized india-rubber	·000089
Horn	·000087
Beeswax	·000087
Felt	·000087
Vulcanite	·0000833
Haircloth	·0000402
Cotton-wool, divided	·0000433
Cotton - wool, pressed	·0000335
Flannel	·0000355
Coarse linen	·0000298
Quartz, along axis	·000922
„	·00124
„	·00057
„	·00083
Quartz, perpendicular to axis	·0040
„	·0044

Professor Forbes quotes a paper by M. Lucien De la Rive (Soc. de Ph. et d'Hist. Nat. de Genève, 1864) in which the following result is obtained for ice,

Ice .... ·00230.

M. De la Rive's experiments are described in 'Annales de Chimie,' sér. 4, tom. i. pp. 504–6.

81. Péclet, in 'Annales de Chimie,' sér. 4, tom. ii. p. 114 [1841], employs as the unit of conductivity the transmission, in one second, through a plate a metre square and a millimetre thick, of as much heat as will raise a cubic decimetre (strictly a kilogramme) of water one degree. Formula (2) shows that the value of this conductivity, in the C.G.S. system, is

$$\frac{1000}{1} \cdot \frac{\frac{1}{10}}{10000}; \text{ that is, } \frac{1}{100}.$$

His results must accordingly be divided by 100; and they then become :—

	Conductivity.		Conductivity.
Gold	·2128	Marble	·0048
Platinum	·2095	Baked earth	·0023
Silver	·2071		
Copper	·1911		
Iron	·0795		
Zinc	·0774		
Lead	·0384		

The value given for lead was from direct experiment. The values given for the other metals were not from direct experiment, but were inferred from the value for lead taken in conjunction with Despretz's results for the relative conductivity of metals.

82. The same author published in 1853 a greatly extended series of observations, in a work entitled "Nouveaux documents relatifs aux chauffage et à la ventilation." In this series, the conductivity which is adopted as unity is the transmission, in one hour, through a plate a metre square and a metre thick, of as much heat as will raise a kilogramme of water one degree. This conductivity, in C.G.S. units, is

$$\frac{1000}{1} \cdot \frac{100}{10000} \cdot \frac{1}{3600}; \text{ that is, } \frac{1}{360}.$$

The results must therefore be divided by 360; and they then become as follows:—

	Density.	Conductivity.
Copper	....	·178
Iron	....	·081
Zinc	....	·078
Lead	....	·039
Carbon from gas-retorts	1·61	·0138
Marble, fine-grained grey	2·68	·0097
„ sugar-white, coarse-grained	2·77	·0077
Limestone, fine-grained	2·34	·0058
„ „	2·27	·0047
„ „	2·17	·0035
Lias building-stone, coarse-grained	2·24	·0037
„ „	2·22	·0035
Plaster of Paris, ordinary, made up	....	·00092
„ very fine „	1·25	·00144
„ for casts, very fine, made up	1·25	·00122
Alum paste (marble-cement) „	1·73	·00175
Terra-cotta	1·98	·00192
„	1·85	·00142
Fir, across fibres	·48	·00026
„ along fibres	·48	·00047
Walnut, across fibres	....	·00029
„ along fibres	....	·00048
Oak, across fibres	....	·00059
Cork	·22	·00029
Caoutchouc	....	·00041
Gutta percha	....	·00048
Starch paste	1·017	·00118
Glass	2·44	·0021
„	2·55	·0024
Sand, quartz	1·47	·00075
Brick, pounded, coarse-grained	1·0	·00039
„ passed through silk sieve	1·76	·00046
Fine brickdust, obtained by decantation	1·55	·00039
Chalk, powdered, slightly damp	·92	·00030
„ washed and dried	·85	·00024
„ washed, dried, and compressed	1·02	·00029
Potato-starch	·71	·00027
Wood-ashes	·45	·00018
Mahogany sawdust	·31	·00018
Wood charcoal, ordinary, powdered	·49	·00022
Bakers' breeze, in powder, passed through silk sieve	·25	·00019
Ordinary wood charcoal, in powder, passed through silk sieve	·41	·000225
Coke, powdered	·77	·00044
Iron filings	2·05	·00044
Binoxide of manganese	1·46	·00045

Woolly Substances.

	Density.	Conductivity.
Cotton-wool, of all densities	....	·000111
Cotton swansdown (molleton de coton), of all densities	....	·000111
Calico, new, of all densities	....	·000139
Wool, carded, of all densities	....	·000122
Woollen swansdown (molleton de laine), of all densities	....	·000067
Eider-down	....	·000108
Hempen cloth, new	·54	·000144
„ old	·58	·000119
Writing-paper, white	·85	·000119
Grey paper, unsized	·48	·000094

Emission and Surface Conduction.

83. Mr. D. M'Farlane has published (Proc. Roy. Soc. 1872, p. 93) the results of experiments on the loss of heat from blackened and polished copper in air at atmospheric pressure. They need no reduction, the units employed being the centimetre, gramme, and second. The general result is expressed by the formulæ

$$x = \cdot 000238 + 3\cdot 06 \times 10^{-6}t - 2\cdot 6 \times 10^{-8}t^2$$

for a blackened surface, and

$$x = \cdot 000168 + 1\cdot 98 \times 10^{-6}t - 1\cdot 7 \times 10^{-8}t^2$$

for polished copper, x denoting the quantity of heat lost per second per square centim. of surface of the copper, per degree of difference between its temperature and that of the walls of the enclosure. These latter were blackened internally, and were kept at a nearly constant temperature of 14° C. The air within the enclosure was kept moist by a saucer of water. The greatest difference of temperature employed in the experiments (in other words, the highest value of t) was 50° or 60° C.

The following Table contains the values of x calculated from the above formulæ, for every fifth degree, within the limits of the experiments.

Difference of Temperature.	Value of x.		Ratio.
	Polished Surface.	Blackened Surface.	
5°	·000178	·000252	·707
10	·000186	·000266	·699
15	·000193	·000279	·692
20	·000201	·000289	·695
25	·000207	·000298	·694
30	·000212	·000306	·693
35	·000217	·000313	·693
40	·000220	·000319	·693
45	·000223	·000323	·690
50	·000225	·000326	·690
55	·000226	·000328	·690
60	·000226	·000328	·690

84. Professor Tait has published (Proc. R. S. E. 1869–70, p. 207) observations by Mr. J. P. Nichol on the loss of heat from blackened and polished copper, in air, at three different pressures, the enclosure being blackened internally and surrounded by water at a temperature of approximately 8° C.* Professor Tait's units are the grain-degree for heat, the square inch for area, and the hour for time. The rate of loss per unit of area is

$$\frac{\text{heat emitted}}{\text{area} \times \text{time}}.$$

The grain-degree is ·0648 gramme-degree.
The square inch is 6·4514 square centims.
The hour is 3600 seconds.
Hence Professor Tait's unit rate of emission is

$$\frac{\cdot 0648}{6\cdot 4514 \times 3600} = 2\cdot 79 \times 10^{-6}$$

of our units. Employing this reducing factor, Professor Tait's Table of results will stand as follows:—

* This temperature is not stated in the '*Proceedings*,' but has been communicated to me by Professor Tait.

Pressure $1{\cdot}014 \times 10^6$ [760 millims. of mercury].

Blackened. Temp. Cent.	Loss per sq. cm. per second.	Bright. Temp. Cent.	Loss per sq. cm. per second.
61°·2	·01746	63°·8	·00987
50·2	·01360	57·1	·00862
41·6	·01078	50·5	·00736
34·4	·00860	44·8	·00628
27·3	·00640	40·5	·00562
20·5	·00455	34·2	·00438
		29·6	·00378
		23·3	·00278
		18·6	·00210

Pressure $1{\cdot}36 \times 10^5$ [102 millims. of mercury].

62·5	·01298	67·8	·00492
57·5	·01158	61·1	·00433
53·2	·01048	55	·00383
47·5	·00898	49·7	·00340
43	·00791	44·9	·00302
28·5	·00490	40·8	·00268

Pressure $1{\cdot}33 \times 10^4$ [10 millims. of mercury].

62·5	·01182	65	·00388
57·5	·01074	60	·00355
54·2	·01003	50	·00286
41·7	·00726	40	·00219
37·5	·00639	30	·00157
34	·00569	23·5	·00124
27·5	·00446		
24·2	·00391		

Mechanical Equivalent of Heat.

85. The value originally deduced by Joule from his experiments on the stirring of water was 772 foot-pounds of work (at Manchester) for as much heat as raises a pound of water through 1° Fahr. This is 1389·6 foot-pounds for a pound of water raised 1° C., or 1389·6 foot-grammes for a gramme of water raised 1° C. As a foot is 30·48 centims., and the value of g at Manchester is 981·3, this is $1389{\cdot}6 \times 30{\cdot}48 \times 981{\cdot}3$ ergs per gramme-degree; that is, $4{\cdot}156 \times 10^7$ ergs per gramme-degree.

A later determination by Joule (Brit. Assoc. Report, 1867, pt. i. p. 522, or 'Reprint of Reports on Electrical Standards,' p. 186) is 25187 foot-grain-second units of work per grain-degree Fahr. This is 45355 of the same units per grain-degree Centigrade, or 45355 foot-gramme-second units of work per gramme-degree Centigrade; that is to say,

$$45355 \times (30{\cdot}48)^2 = 4{\cdot}214 \times 10^7$$

ergs per gramme-degree Centigrade.

Some of the best determinations by various experimenters are given (in gravitation measure) in the following list, extracted from Watts's 'Dictionary of Chemistry,' Supplement 1872, p. 687. The value 429·3 in this list corresponds to $4{\cdot}214 \times 10^7$ ergs:—

Hirn	432	Friction of water and brass.
"	433	Escape of water under pressure.
"	441·6	Specific heats of air.
"	425·2	Crushing of lead.
Joule	429·3	Heat produced by an electric current.
Violle	435·2 (copper) .... 434·9 (aluminium) . 435·8 (tin) .... 437·4 (lead)	Heat produced by induced currents.
Regnault	437	Velocity of sound.

We shall adopt $4{\cdot}2 \times 10^7$ ergs as the equivalent of 1 gramme-degree; that is, employing J as usual to denote Joule's equivalent, we have

$$J = 4{\cdot}2 \times 10^7 = 42 \text{ millions.}$$

86. *Heat and Energy of Combination with Oxygen.*

1 gramme of	Compound formed.	Gramme-degrees of heat produced.	Equivalent Energy, in ergs.
Hydrogen	H^2O	34000 A F	$1{\cdot}43 \times 10^{12}$
Carbon	CO^2	8000 A F	$3{\cdot}36 \times 10^{11}$
Sulphur	SO^2	2300 A F	$9{\cdot}66 \times 10^{10}$
Phosphorus	P^2O^5	5747 A	$2{\cdot}41 \times 10^{11}$
Zinc	ZnO	1301 A	$5{\cdot}46 \times 10^{10}$
Iron	Fe^3O^4	1576 A	$6{\cdot}62 \times 10^{10}$
Tin	SnO^2	1233 A	5·18 "
Copper	CuO	602 A	2·53 "
Carbonic oxide ..	CO^2	2420 A F	$1{\cdot}02 \times 10^{11}$
Marsh-gas	CO^2 and H^2O	13100 A F	5·50 "
Olefiant gas	"	11900 A F	5·00 "
Alcohol	"	6900 A F	2·90 "

Combustion in Chlorine.

Hydrogen	HCl	23000 F T	$9{\cdot}66 \times 10^{11}$
Potassium	KCl	2655 A	1·12 "
Zinc	$ZnCl^2$	1529 A	$6{\cdot}42 \times 10^{10}$
Iron	Fe^2Cl^6	1745 A	7·33 "
Tin	$SnCl^4$	1079 A	4·53 "
Copper	$CuCl^2$	961 A	4·04 "

The numbers in the last column are the products of the numbers in the preceding column by 42 millions.

The authorities for these determinations are indicated by the initial letters A (Andrews), F (Favre and Silbermann), T (Thomsen). Where two initial letters are given, the number adopted is intermediate between those obtained by the two experimenters.

87. Difference between the *two specific heats* of a gas.

Let s_1 denote the specific heat of a given gas at constant pressure,
s_2 the specific heat of a given gas at constant volume,
α the coefficient of expansion per degree,
v the volume of 1 gramme of the gas in cubic centim. at pressure p dynes per square centim.

When a gramme of the gas is raised from 0° to 1° at the constant pressure p, the heat taken in is s_1, the increase of volume is αv, and the work done against external resistance is αvp (ergs). This work is the equivalent of the difference between s_1 and s_2; that is, we have

$$s_1 - s_2 = \frac{\alpha vp}{\mathrm{J}}, \text{ where } \mathrm{J} = 4{\cdot}2 \times 10^7.$$

For dry air at 0° the value of vp is $7{\cdot}838 \times 10^8$, and α is ·003665. Hence we find $s_1 - s_2 = {\cdot}0684$. The value of s_1, according to Regnault, is ·2375. Hence the value of s_2 is ·1691.

The value of $\frac{s_1 - s_2}{v}$, or $\frac{\alpha p}{\mathrm{J}}$, for dry air at 0° and a megadyne per square centim. is

$$\frac{s_1 - s_2}{v} = \frac{{\cdot}0684}{783{\cdot}8} = 8{\cdot}726 \times 10^{-5};$$

and this is also the value of $\frac{s_1 - s_2}{v}$ for any other gas (at the same temperature and pressure) which has the same coefficient of expansion.

88. *Change of freezing-point* due to change of *pressure*.

Let the volume of the substance in the liquid state be to its volume in the solid state as 1 to $1 + e$.

When unit volume in the liquid state solidifies under pressure $P+p$, the work done by the substance is the product of $P+p$ by the increase of volume e, and is therefore $Pe+pe$.

If it afterwards liquefies under pressure P, the work done against the resistance of the substance is Pe; and if the pressure be now increased to $P+p$, the substance will be in the same state as at first.

Let T be the freezing temperature at pressure P,
$T+t$ the freezing temperature at pressure $P+p$,
l the latent heat of liquefaction,
d the density of the liquid.

Then d is the mass of the substance, and ld is the heat taken in at the temperature of melting T. Hence, by thermodynamic principles, the heat converted into mechanical effect in the cycle of operations is

$$\frac{-t}{T+273} \cdot ld\,.$$

But the mechanical effect is pe. Hence we have

$$\frac{-t}{T+273}ld=\frac{pe}{J}$$

$$-\frac{t}{p}=\frac{e(T+273)}{Jld}: \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

$-\frac{t}{p}$ is the lowering of the freezing-point for an additional pressure of a dyne per square centim.; and $-\frac{t}{p}\times 10^6$ will be the lowering of the freezing-point for each additional atmosphere of 10^6 dynes per square centim.

For water we have

$$e=\cdot 087,\ l=79\cdot 25,\ T=0,\ d=1$$

$$-\frac{t}{p}\times 10^6=\frac{\cdot 087\times 273}{42\times 79\cdot 25}=\cdot 00714.$$

Formula (3) shows that $\frac{t}{p}$ is opposite in sign to e. Hence the freezing-point will be raised by pressure if the substance contracts in solidifying.

89. Change of *temperature* produced by *adiabatic compression* of a fluid; that is, by compression under such circumstances that no heat enters or leaves the fluid.

Let a cubic centim. of fluid at the initial temperature t° C. and pressure p dynes per square centim. be subjected to the following cycle of four operations:—

1. Increase of pressure, adiabatically, from p to $p+\varpi$, ϖ being small.

2. Addition of heat, at constant pressure $p+\varpi$, till the temperature rises by the amount dt.

3. Diminution of pressure, adiabatically, from $p+\varpi$ to p.

4. Subtraction of heat, at constant pressure p, till the temperature falls to t.

Let τ denote the increase of temperature, and v the diminution of volume in (1); and let e denote the expansion per degree at constant pressure.

Then, neglecting small quantities of the second order, the changes of pressure, temperature, and volume are as shown in the following tabular statement:—

Operation.	Pressure.	Temperature.	Change of volume.
1	p to $p+\varpi$	t to $t+\tau$	$-v$
2	$p+\varpi$	$t+\tau$ to $t+\tau+dt$	edt
3	$p+\varpi$ to p	$t+\tau+dt$ to $t+dt$	v
4	p	$t+dt$ to t	$-edt$

The work done by the fluid in the operations (1) and (3), taken together, is zero.

The work done by the fluid in the operations (2) and (4), taken together, is ϖedt.

The heat taken in by the fluid in (2) is, as far as small quantities of the first order are concerned, equal to that given out in (4), and is Cdt, C denoting the thermal capacity of a cubic centim. of the substance at constant pressure.

If T denote the absolute temperature, or $273+t$, the heat converted into mechanical effect is $\frac{\tau}{T}Cdt$; and this must be equal to $\frac{\varpi edt}{J}$. We have therefore $\frac{\tau}{T}C=\frac{\varpi e}{J}$, or $\tau=\frac{T\varpi e}{JC}$, where τ

denotes the increase of temperature produced by the increase ϖ of pressure.

90. *Elasticity* as affected by *heat of compression*.

The expansion due to the increase of temperature τ, above calculated, is τe; that is, $\frac{T\varpi e^2}{JC}$; and the ratio of this expansion to the contraction $\frac{\varpi}{E}$, which would be produced at constant temperature (E denoting the elasticity of volume at constant temperature), is $\frac{ETe^2}{JC}:1$. Putting m for $\frac{ETe^2}{JC}$, the elasticity for adiabatic compression will be $\frac{E}{1-m}$; or, if m is small, $E(1+m)$; and this value is to be used instead of E in calculating the change of volume due to sudden compression.

The same formula expresses the value of Young's modulus of elasticity for sudden extension or compression of a solid in one direction, E now denoting the value of the modulus at constant temperature.

Examples.

For compression of water between 10° and 11° we have

$$E=2{\cdot}1\times10^{10},\quad T=283,\quad e={\cdot}000092,\quad C=1;$$

hence

$$\frac{ETe^2}{JC}={\cdot}0012.$$

For longitudinal extension of iron at 10° we have

$$E=1{\cdot}96\times10^{12},\quad T=283,\quad e={\cdot}0000122,\quad C={\cdot}109\times7{\cdot}7;$$

hence

$$\frac{ETe^2}{JC}={\cdot}00234.$$

Thus the heat of compression increases the volume-elasticity of water at this temperature by about $\frac{1}{8}$ per cent., and the longitudinal elasticity of iron by about $\frac{1}{4}$ per cent.

For dry air at 0° and a megadyne per square centim. we have

$$E=10^6,\quad T=273,\quad e=\frac{1}{273},\quad C={\cdot}2375\times{\cdot}001276,$$

$$m = \frac{ETe^2}{JC} = \cdot 288, \quad \frac{1}{1-m} = 1\cdot 404.$$

91. *Expansions of Volume per degree Cent.* (*abridged from Watts's 'Dictionary of Chemistry,' Article Heat*, pp. 67, 68, 71).

Glass	·00002	to	·00003
Iron	·000035	„	·000044
Copper	·000052	„	·000057
Platinum	·000026	„	·000029
Lead	·000084	„	·000089
Tin	·000058	„	·000069
Zinc	·000087	„	·000090
Gold	·000044	„	·000047
Brass	·000053	„	·000056
Silver	·000057	„	·000064
Steel	·000032	„	·000042
Cast iron........ about	·000033		

These results are partly from direct observation, and partly calculated from observed linear expansion.

Expansion of Mercury, according to Regnault (*Watts's Dictionary*, p. 56).

Temp. = t. °	Volume at t.	Expansion per degree at t°.
0	1·000000	·00017905
10	1·001792	17950
20	1·003590	18001
30	1·005393	18051
50	1·009013	18152
70	1·012655	18253
100	1·018153	18405

The temperatures are by air-thermometer.

Expansion of Alcohol and Ether, according to Kopp (*Watts's Dictionary*, p. 62).

Temp. °	Volume. Alcohol.	Ether.
0	1·0000	1·0000
10	1·0105	1·0152
20	1·0213	1·0312
30	1·0324	1·0483
40	1·0440	1·0667

CHAPTER X.

MAGNETISM.

92. The *unit* magnetic *pole,* or the pole of *unit strength,* is that which repels an equal pole at unit distance with unit force. In the C.G.S. system it is the pole which repels an equal pole, at the distance of 1 centimetre, with a force of 1 dyne.

If P denote the strength of a pole, it will repel an equal pole at the distance L with the force $\frac{P^2}{L^2}$. Hence we have the dimensional equations

$$P^2L^{-2} = \text{force} = MLT^{-2},\ P^2 = ML^3T^{-2},\ P = M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1};$$

that is, the dimensions of a *pole* (or the dimensions of *strength of pole*) are $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$.

93. The *intensity* of a magnetic *field* is the force which a unit pole will experience when placed in it. Denoting this intensity by I, the force on a pole P will be IP. Hence

$$IP = \text{force} = MLT^{-2},\ I = MLT^{-2}.\ M^{-\frac{1}{2}}L^{-\frac{3}{2}}T = M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1};$$

that is, the dimensions of *field-intensity* are $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$.

94. The *moment* of a *magnet* is the product of the strength of either of its poles by the distance between them. Its dimensions are therefore LP; that is, $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$.

Or, more rigorously, the moment of a magnet is a quantity which, when multiplied by the intensity of a uniform field, gives the couple which the magnet experiences when held with its axis perpendicular to the lines of force in this field. It is therefore the quotient of a couple ML^2T^{-2} by a field-intensity $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$; that is, it is $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$ as before.

95. If different portions be cut from a uniformly magnetized substance, their moments will be simply as their volumes. Hence the *intensity* of *magnetization* of a uniformly magnetized body is defined as the quotient of its moment by its volume. But we have

$$\frac{\text{moment}}{\text{volume}} = M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}\ .\ L^{-3} = M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}.$$

Hence *intensity* of *magnetization* has the same dimensions as *intensity* of *field*. When a magnetic substance (whether paramagnetic or diamagnetic) is placed in a magnetic field, it is magnetized by induction; and each substance has its own specific *coefficient of magnetic induction* (constant, or nearly so, when the field is not excessively intense), which expresses the ratio of the intensity of the induced magnetization to the intensity of the field. For paramagnetic substances (such as iron) this coefficient is positive; for diamagnetic substances (such as bismuth) it is negative; that is to say, the induced polarity is reversed end for end as compared with that of a paramagnetic substance placed in the same field.

96. The work required to move a pole P from one point to another is the product of P by the difference of the *magnetic potentials* of the two points. Hence the dimensions of magnetic potential are

$$\frac{\text{work}}{P} = ML^2T^{-2} \cdot M^{-\frac{1}{2}}L^{-\frac{3}{2}}T = M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}.$$

Examples.

1. To find the multiplier for reducing magnetic intensities from the foot-grain-second system to the C.G.S. system.

The dimensions of the unit of intensity are $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}$. In the present case we have $M = \cdot 0648$, $L = 30 \cdot 48$, $T = 1$, since a grain is ·0648 gramme, and a foot is 30·48 centim. Hence $M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} = \sqrt{\frac{\cdot 0648}{30 \cdot 48}} = \cdot 04611$; that is, the foot-grain-second unit of intensity is denoted by the number ·04611 in the C.G.S. system. This number is accordingly the required multiplier.

2. To find the multiplier for reducing intensities from the millimetre-milligramme-second system to the C.G.S. system, we have

$$M = \frac{1}{1000}, \quad L = \frac{1}{10}, \quad T = 1, \quad M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1} = \sqrt{\frac{10}{1000}} = \frac{1}{10}.$$

Hence $\frac{1}{10}$ is the required multiplier.

3. Gauss states (Taylor's 'Scientific Memoirs,' vol. ii. p. 225) that the magnetic moment of a steel bar-magnet, of one pound weight, was found by him to be 100877000 millimetre-milligramme-second units. Find its moment in C.G.S. units.

Here the value of the unit moment employed is, in terms of C.G.S units, $M^{\frac{1}{2}}L^{\frac{5}{2}}T^{-1}$, where M is 10^{-3}, L is 10^{-1}, and T is 1; that is, its value is $10^{-\frac{3}{2}} . 10^{-\frac{5}{2}} = 10^{-4}$. Hence the moment of the bar is 10087·7 C.G.S. units.

4. Find the mean intensity of magnetization of the bar, assuming its specific gravity to be 7·85, and assuming that the pound mentioned in the question is the pound avoirdupois of 453·6 grammes.

Its mass in grammes, divided by its density, will be its volume in cubic centimetres; hence we have

$$\frac{453 \cdot 6}{7 \cdot 85} = 57 \cdot 78 = \text{volume of bar.}$$

$$\text{Intensity of magnetization} = \frac{\text{moment}}{\text{volume}} = \frac{10088}{57 \cdot 78} = 174 \cdot 6$$

5. Kohlrausch states ('Physical Measurements,' p. 195, English edition) that the *maximum* of *permanent magnetism* which very thin rods can retain is about 1000 millimetre-milligramme-second units of moment for each milligramme of steel. Find the corresponding moment per gramme in C.G.S. units, and the corresponding intensity of magnetization.

For the moment of a milligramme we have $1000 \times 10^{-4} = 10^{-1}$.

For the volume of a milligramme we have $(7 \cdot 85)^{-1} \times 10^{-3}$, taking 7·85 as the density of steel.

Hence the moment per gramme is $10^{-1} \times 10^{3} = 100$, and the intensity of magnetization is $100 \times 7 \cdot 85 = 785$.

6. The *maximum intensity* of magnetization for specimens of iron, steel, nickel, and cobalt has been determined by Professor Rowland (Phil. Mag. 1873, vol. xlvi. p. 157, and November 1874)—that is to say, the limit to which their intensities of magnetization would approach, if they were employed as the cores of electromagnets, and the strength of current and number of convolutions if the coil were indefinitely increased. Professor

Rowland's fundamental units are the metre, gramme, and second; hence his unit of intensity is $\frac{1}{10}$ of the C.G.S. unit. His values, reduced to C.G.S. units, are

Iron	1390 to 6930
Steel	416 „ 1610
Nickel	212 „ 383
Cobalt	800

7. Gauss states (*loc. cit.*) that the magnetic *moment of the earth*, in millimetre-milligramme-second measure, is

$$3{\cdot}3092\,R^3,$$

R denoting the earth's radius in millimetres. Reduce this value to C.G.S.

Since R^3 is of the dimensions of volume, the other factor, 3·3092, must be of the dimensions of intensity. Hence, employing the reducing factor 10^{-1} above found, we have ·33092 as the corresponding factor for C.G.S. measure; and the moment of the earth will be

$$\cdot 33092\,R^3,$$

R denoting the earth's radius in centimetres—that is, $6{\cdot}37 \times 10^8$.

We have

$$\cdot 33092 \times (6{\cdot}37 \times 10^8)^3 = 8{\cdot}55 \times 10^{25}$$

for the *earth's magnetic moment* in C.G.S. units.

8. From the above result, deduce the intensity of magnetization of the earth regarded as a uniformly magnetized body.

We have

$$\text{intensity} = \frac{\text{moment}}{\text{volume}} = \frac{8{\cdot}55 \times 10^{25}}{1{\cdot}083 \times 10^{27}} = \cdot 0790.$$

This is about $\frac{1}{2200}$ of the intensity of magnetization of Gauss's pound magnet; so that 2·2 cubic decimetres of earth would be equivalent to 1 cubic centim. of strongly magnetized steel, if the observed effects of terrestrial magnetism were due to uniform magnetization of the earth's substance.

9. Gauss, in his papers on terrestrial magnetism, employs two different units of intensity, and makes mention of a third as "the unit in common use." The relation between them is pointed out in the passage above referred to. The total intensity at Göttingen, for the 19th of July, 1834, was 4·7414 when expressed in terms of one of these units—the millimetre-

milligramme-second unit; was 1357 when expressed in terms of the other unit employed by Gauss, and 1·357 in terms of the "unit in common use." In C.G.S. measure it would be ·47414.

97. A first approximation to the *distribution* of magnetic force *over the earth's surface* is obtained by assuming the earth to be uniformly magnetized, or, what is mathematically equivalent to this, by assuming the observed effects to be due to a small magnet at the earth's centre. The moment of the earth on the former supposition, or the moment of the small magnet on the latter, must be

$$\cdot 33092\, R^3,$$

R denoting the earth's radius in centims. The magnetic poles, on these suppositions, must be placed at

77° 50′ north lat., 296° 29′ east long.,

and at 77° 50′ south lat., 116° 29′ east long.

The intensity of the horizontal component of terrestrial magnetism, at a place distant A° from either of these poles, will be

$$\cdot 33092 \sin A^\circ;$$

the intensity of the vertical component will be

$$\cdot 66184 \cos A^\circ;$$

and the tangent of the dip will be

$$2 \cot A^\circ.$$

The magnetic potential, on the same suppositions, will be

$$\cdot 33092\, R \cos A^\circ;$$

see Maxwell, 'Electricity and Magnetism,' vol. ii. p. 8. Gauss's approximate expression for the potential and intensity at an arbitrary point on the earth's surface consists of four successive approximations, of which this is the first.

98. The mean value of the intensity of *horizontal magnetic force* at Greenwich was

·1716 in the year 1848,
·1776 „ 1867;

and its rate of increase in successive years is sensibly uniform.

The place of *greatest horizontal intensity* is in lat. 0°, long. 259° E., where the value is ·3733.

In 1843 the *dip* at Greenwich was about 69° 1′; it has diminished, with a rate continually accelerating, till in 1868 it was 67° 56′. The *total intensity*, as computed from the dip and horizontal intensity, was

·4791 in the year 1848,
·4740 „ 1866.

The place of *greatest total intensity* is in South Victoria, about 70° S., 160° E., where its value is ·7898.

The place of *least total intensity* is near St. Helena, in lat. 16° S., long. 355° E., where its value is ·2828.

We have extracted these data from 'Airy on Magnetism,' pp. 74, 93, 94, 97, & 98.

99. In the 'Report of the Astronomer Royal to the Board of Visitors,' June 1874, the following data are given:—

Mean westerly declination for 1873..	19° 30′ nearly.
Mean horizontal force for 1873	3·883 (in English units). 1·791 (in metric units).
Mean dip for 1873................	67° 43′ 52″ (by 9-inch needles). 67 45 46 (by 6-inch needles). 67 47 17 (by 3-inch needles).
[Mean..................	67 45 38]

[The "English units" are foot-grain-second units. The "metric units" are metre-gramme-second units. The value of the horizontal force in C.G.S. units is ·1791.]

In De La Rue's 'Diary' for 1875 the following are given as the estimated values of magnetic elements at the Royal Observatory, Greenwich, for 1875:—

Declination......	19° 16′ W.	Dip 67° 42′
Horizontal force..	3·89	................English units.
Vertical force....	9·49	

Reduced to C.G.S. measure, these latter values are:—

Horizontal force............	·1794
Vertical force...............	·4376

CHAPTER XI.

ELECTRICITY.

Electrostatics.

100. If q denote the numerical value of a *quantity* of electricity in electrostatic measure, the mutual force between two equal quantities q at the mutual distance l will be $\frac{q^2}{l^2}$. In the C.G.S. system the electrostatic unit of electricity is accordingly that quantity which would repel an equal quantity at the distance of 1 centim. with a force of 1 dyne.

Since the dimensions of force are $\frac{ml}{t^2}$, we have, as regards dimensions,

$$\frac{q^2}{l^2}=\frac{ml}{t^2},\ \text{whence}\ q^2=\frac{ml^3}{t^2},\ q=m^{\frac{1}{2}}l^{\frac{3}{2}}t^{-1}.$$

101. The work done in raising a quantity of electricity q through a difference of *potential* v is qv.

Hence we have

$$v=\frac{\text{work}}{q}=ml^2t^{-2}\ .\ m^{-\frac{1}{2}}l^{-\frac{3}{2}}t=m^{\frac{1}{2}}l^{\frac{1}{2}}t^{-1}.$$

In the C.G.S. system the unit difference of potential is that difference through which a unit of electricity must be raised that the work done may be 1 erg.

Or, we may define potential as the quotient of quantity of electricity by distance. This gives

$$v=m^{\frac{1}{2}}l^{\frac{3}{2}}t^{-1}\ .\ l^{-1}=m^{\frac{1}{2}}l^{\frac{1}{2}}t^{-1},\ \text{as before.}$$

In the C.G.S. system the unit of potential is the potential due to unit quantity at the distance of 1 centim.

102. The *capacity* of a conductor is the quotient of the quantity of electricity with which it is charged by the potential which this charge produces in it. Hence we have

$$\text{capacity}=\frac{q}{v}=m^{\frac{1}{2}}l^{\frac{3}{2}}t^{-1}\ .\ m^{-\frac{1}{2}}l^{-\frac{1}{2}}t=l.$$

The same conclusion might have been deduced from the fact

that the capacity of an isolated spherical conductor is equal (in numerical value) to its radius. The C.G.S. unit of capacity is the capacity of an isolated sphere of 1 centim. radius.

103. The numerical value of a *current* (or the strength of a current) is the quantity of electricity that passes in unit time. Hence the dimensions of *current* are $\frac{q}{t}$; that is, $m^{\frac{1}{2}}l^{\frac{3}{2}}t^{-2}$.

The C.G.S. unit of current is that current which conveys the above defined unit of quantity in 1 second.

104. The dimensions of *resistance* can be deduced from Ohm's law, which asserts that the resistance of a wire is the quotient of the difference of potential of its two ends, by the current which passes through it. Hence we have

$$\text{resistance} = m^{\frac{1}{2}}l^{\frac{1}{2}}t^{-1} \,.\, m^{-\frac{1}{2}}l^{-\frac{3}{2}}t^{2} = l^{-1}t.$$

Or, the resistance of a conductor is equal to the time required for the passage of a unit of electricity through it, when unit difference of potential is maintained between its ends. Hence

$$\text{resistance} = \frac{\text{time} \times \text{potential}}{\text{quantity}} = t \,.\, m^{\frac{1}{2}}l^{\frac{1}{2}}t^{-1} \,.\, m^{-\frac{1}{2}}l^{-\frac{3}{2}}t = l^{-1}t.$$

105. As the force upon a quantity q of electricity, in a field of electrical force of intensity i, is iq, we have

$$i = \frac{\text{force}}{q} = mlt^{-2} \,.\, m^{-\frac{1}{2}}l^{-\frac{3}{2}}t = m^{\frac{1}{2}}l^{-\frac{1}{2}}t^{-1}.$$

The quantity here denoted by i is commonly called the "electrical force at a point."

Electromagnetics.

106. A *current* C (or a current of *strength* C) flowing along a circular arc produces at the centre of the circle an intensity of magnetic field equal to C multiplied by length of arc divided by square of radius. Hence C divided by a length is equal to a field-intensity, or

$$\mathrm{C} = \text{length} \times \text{intensity} = \mathrm{L} \,.\, \mathrm{M}^{\frac{1}{2}}\mathrm{L}^{-\frac{1}{2}}\mathrm{T}^{-1} = \mathrm{L}^{\frac{1}{2}}\mathrm{M}^{\frac{1}{2}}\mathrm{T}^{-1}.$$

107. The *quantity* of electricity Q conveyed by a current is

the product of the current by the time that it lasts. The dimensions of Q are therefore $L^{\frac{1}{2}}M^{\frac{1}{2}}$.

108. The work done in urging a quantity Q through a circuit, by an *electromotive force* E, is EQ; and the work done in urging a quantity Q through a conductor, by means of a *difference of potential* E between its ends, is EQ. Hence the dimensions of electromotive force, and also the dimensions of *potential*, are $ML^2T^{-2} . L^{-\frac{1}{2}}M^{-\frac{1}{2}}$, or $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$.

109. The *capacity* of a conductor is the quotient of quantity of electricity by potential. Its dimensions are therefore

$$M^{\frac{1}{2}}L^{\frac{1}{2}} . M^{-\frac{1}{2}}L^{-\frac{3}{2}}T^2; \text{ that is, } L^{-1}T^2.$$

110. Resistance is $\frac{E}{C}$; its dimensions are therefore

$$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2} . M^{-\frac{1}{2}}L^{-\frac{1}{2}}T; \text{ that is, } LT^{-1}.$$

111. The following Table exhibits the dimensions of each electrical element in the two systems, together with their ratios:—

	Dimensions in electrostatic system.	Dimensions in electromagnetic system.	$\frac{\text{Dimensions in E.S.}}{\text{Dimensions in E.M.}}$
Quantity	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}$	LT^{-1}
Current	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	LT^{-1}
Capacity	L	$L^{-1}T^2$	L^2T^{-2}
Potential and electromotive force .	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$	$L^{-1}T$
Resistance	$L^{-1}T$	LT^{-1}	$L^{-2}T^2$

112. The *heat generated* in time T by the passage of a current C through a wire of resistance R (when no other work is done by the current in the wire) is $\frac{C^2RT}{J}$ gramme-degrees, J denoting $4{\cdot}2 \times 10^7$; and this is true whether C and R are expressed in electromagnetic or in electrostatic units.

Ratios of the two sets of Electric Units.

113. Let us consider any general system of units based on

a unit of length equal to L centims.,
a unit of mass equal to M grammes,
a unit of time equal to T seconds.

Then we shall have the electrostatic unit of quantity equal to

$$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1} \text{ C.G.S. electrostatic units of quantity,}$$

and the electromagnetic unit of quantity equal to

$$M^{\frac{1}{2}}L^{\frac{1}{2}} \text{ C.G.S. electromagnetic units of quantity.}$$

It is possible so to select L and T that the electrostatic unit of quantity shall be equal to the electromagnetic unit. We shall then have (dividing out by $M^{\frac{1}{2}}L^{\frac{1}{2}}$).

LT^{-1} C.G.S. electrostatic units $=1$ C.G.S. electromagnetic unit;

or the ratio of the C.G.S. electromagnetic unit to the C.G.S. electrostatic unit is $\frac{L}{T}$.

Now $\frac{L}{T}$ is clearly the value in centims. per second of that velocity which would be denoted by unity in the new system. This is *a definite concrete velocity*; and its numerical value will always be equal to the ratio of the electromagnetic to the electrostatic unit of quantity, whatever units of length, mass, and time are employed.

114. It will be observed that the ratio of the two units of quantity is the inverse ratio of their dimensions; and the same can be proved in the same way of the other four electrical elements. The last column of the above Table shows that M does not enter into any of the ratios, and that L and T enter with equal and opposite indices, showing that all the ratios depend only on the velocity $\frac{L}{T}$.

Thus, if the concrete velocity $\frac{L}{T}$ be a velocity of v centims

per second, the following relations will subsist between the C.G.S. units :—

1 electromagnetic unit of	quantity	$=v$	electrostatic units.
1 „ „	current	$=v$	„
1 „ „	capacity	$=v^2$	„
v electromagnetic units of	potential	$=1$	electrostatic unit.
v^2 „ „	resistance	$=1$	„

115. Weber and Kohlrausch, by an experimental comparison of the two units of quantity, determined the value of v to be

$$3{\cdot}1074 \times 10^{10} \text{ centims. per second.}$$

Sir W. Thomson, by an experimental comparison of the two units of potential, determined the value of v to be

$$2{\cdot}825 \times 10^{10}.$$

Prof. Clerk Maxwell, by an experiment in which an electrostatic attraction was balanced by an electrodynamic repulsion, determined the value of v to be

$$2{\cdot}88 \times 10^{10}.$$

All these determinations differ but little from the velocity of light in vacuo, which, according to Foucault's determination, is

$$2{\cdot}98 \times 10^{10},$$

and according to the recent experiments of Cornu (see 'Nature,' February 4, 1875, p. 275) is

$$3{\cdot}004 \times 10^{10}.$$

We shall adopt the round number

$$3 \times 10^{10}$$

as the value of v.

116. The dimensions of the electric units are rather simpler when expressed in terms of length, *density*, and time.

Putting D for density, we have $M = L^3D$. Making this substitution for M, in the expressions above obtained (§ 111), we have the following results :—

	Electrostatic.		Electromagnetic.
Quantity	$D^{\frac{1}{2}}L^{3}T^{-1}$	......	$D^{\frac{1}{2}}L^{2}$
Current	$D^{\frac{1}{2}}L^{3}T^{-2}$	......	$D^{\frac{1}{2}}L^{2}T^{-1}$
Capacity	L	......	$L^{-1}T^{2}$
Potential....	$D^{\frac{1}{2}}L^{2}T^{-1}$	......	$D^{\frac{1}{2}}L^{3}T^{-2}$
Resistance ..	$L^{-1}T$	......	LT^{-1}

It will be noted that the exponents of L and T in these expressions are free from fractions.

Ohm, Volt, &c.

117. The unit of resistance employed by practical electricians is the *Ohm*, which is defined by certain standard coils, each of which is to be taken at a stated temperature. The resistance of each of these coils at its proper temperature is intended to be

$$10^9 \text{ C.G.S. electromagnetic units of resistance.}$$

We shall therefore speak of 10^9 C.G.S. units as the *theoretical ohm*. The practical ohm was constructed under the direction of a Committee of the British Association, its construction being based upon experiments in which the resistance of a certain coil of wire was determined in electromagnetic measure.

F. Kohlrausch has since conducted experiments (see Phil. Mag. 1874, vol. xlvii.) from which he infers that the practical ohm (as defined by the standard coils) is

$$1{\cdot}0196 \times 10^9 \text{ C.G.S. units;}$$

and still more recently Lorenz (Pogg. Ann. 1873, vol. cxlix. p. 251) has determined its value to be

$$\cdot 9797 \times 10^9 \text{ C.G.S. units}*.$$

* Lorenz's result is

$$1 \text{ Siemens unit} = \cdot 9337 \times 10^9 \text{ C.G.S. units.}$$

Taking, with Kohlrausch, the practical ohm as equal to 1·0493 Siemens, we have

$$1{\cdot}0493 \times \cdot 9337 = \cdot 9797.$$

118. The practical unit of electromotive force is the *Volt.* Its theoretical value is

$$1 \text{ Volt} = 10^8 \text{ C.G.S. units of potential.}$$

The practical unit of current is the current due to an electromotive force of 1 volt working through a resistance of 1 ohm. It is sometimes called the *Weber.* We have

$$1 \text{ Weber} = \frac{10^8}{10^9} = \frac{1}{10} \text{ of C.G.S. unit of current.}$$

The practical unit of quantity has not received any distinct name. It is the quantity conveyed by a Weber in a second, and is equal to

$$\frac{1}{10} \text{ of C.G.S. unit of quantity.}$$

The capacity of a condenser which holds this quantity when charged to a potential of 1 volt is called the *Farad*

$$= \frac{10^{-1}}{10^8} = 10^{-9} \text{ C.G.S. unit of capacity.}$$

As the farad is much too large for practical convenience, its millionth part, called the *microfarad,* is practically employed; and standard condensers are in use which are guaranteed to be of this capacity. We have

$$1 \text{ microfarad} = 10^{-15} \text{ C.G.S. unit of capacity.}$$

119. By way of assisting the memory, it is useful to remark that the numerical value of the *ohm* is the same as the numerical value of a velocity of one *earth-quadrant per second,* since the length of a quadrant of the meridian is 10^9 centims. This equality will subsist whatever fundamental units are employed, since the dimensions of resistance are the same as the dimensions of velocity.

No special names have as yet been assigned to any electrostatic units.

Electric Spark.

120. Intensity of electric force between two parallel conducting surfaces requisite that a spark may pass between them :—

Distance between surfaces.	Intensity of force.	Difference of potential between surfaces.	
		In electrostatic units.	In electromagnetic units.
·0086	267·1	2·30	$6{\cdot}90 \times 10^{10}$
·0127	257·0	3·26	9·78 „
·0127	262·2	3·33	9·99 „
·0190	224·2	4·26	12·78 „
·0281	200·6	5·64	16·92 „
·0408	151·5	6·18	18·54 „
·0563	144·1	8·11	24·33 „
·0584	139·6	8·15	24·45 „
·0688	140·8	9·69	29·07 „
·0904	134·9	12·20	36·60 „
·1056	132·1	13·95	41·85 „
·1325	131·0	17·36	52·08 „

The first two columns of the above Table are extracted from a paper by Sir W. Thomson (p. 258 of 'Collected Papers'). The numbers in the third column are the products of those in the first and second, intensity of electric force being the same thing as the rate of variation of potential *per unit of distance*, and being in the present case uniform across the whole distance between the surfaces. The numbers in the fourth column are the products of the numbers in the third by 3×10^{10}.

121. The electromotive force of a Daniell's cell is

·00374 electrostatic unit,

as determined by Sir W. Thomson from observation of the attraction between two parallel disks connected with the opposite poles of a Daniell's battery (p. 245 of 'Collected Papers').

122. The resistance of a wire (or more generally of a prism or cylinder) of given material varies directly as its length, and inversely as its cross section. It is therefore equal to

$$R \frac{\text{length}}{\text{section}},$$

where R is a coefficient depending only on the material. R is called the *specific resistance* of the material. Its reciprocal $\frac{1}{R}$ is called the *specific conductivity* of the material.

R is obviously the resistance between two opposite faces of a unit cube of the substance. Hence in the C.G.S. system it is the resistance between two opposite faces of a cubic centim. (supposed to have the form of a cube).

The dimensions of specific resistance are resistance × length; that is, in electromagnetic measure, velocity × length; that is, L^2T^{-1}.

Resistance.

123. *Table of Specific Resistances, in electromagnetic measure (at 0° C. unless otherwise stated).*

	Specific resistance.	Percentage variation per degree, at 20° C.	Specific gravity.
Silver, hard-drawn	1609	·377	10·50
Copper, "	1642	·388	8·95
Gold, "	2154	·365	19·27
Lead, pressed	19847	·387	11·391
Mercury, liquid	96146	·072	13·595
Gold 2, silver 1, hard or annealed	10988	·065	15·218
Selenium at 100° C., crystalline	6×10^{13}	1·00	
Water at 22° C.	$7·18 \times 10^{10}$	·47	
" with ·2 per cent. H_2SO_4	4·47 "	·47	
" " 8·3 " "	$3·32 \times 10^{9}$	·653	
" " 20 " "	1·44 "	·799	
" " 35 " "	1·26 "	1·259	
" " 41 " "	1·37 "	1·410	
Sulphate of zinc and water, $ZnSO_4 + 23H_2O$ at 23° C.	$1·87 \times 10^{10}$		
Sulphate of copper and water, $CuSO_4 + 45H_2O$ at 22° C.	1·95 "		
Glass at 200° C.	$2·27 \times 10^{16}$		
" 250°	$1·39 \times 10^{15}$		
" 300°	$1·48 \times 10^{14}$		
" 400°	$7·35 \times 10^{13}$		
Gutta percha at 24° C.	$3·53 \times 10^{23}$		
" 0° C.	7×10^{24}		

For the authorities for the above numbers see Maxwell, 'Electricity and Magnetism,' vol. i. last chapter.

124. The following Table of specific resistances of metals at

0° C. is reduced from Table IX. in Jenkin's Cantor Lectures. It is based on Matthiessen's experiments.

	Specific resistance.	Percentage of variation for a degree, at 20° C.
Silver, annealed	1521	·377
„ hard-drawn	1652	
Copper, annealed	1615	·388
„ hard-drawn	1652	
Gold, annealed	2081	·365
„ hard-drawn	2118	
Aluminium, annealed	2946	
Zinc, pressed	5690	·365
Platinum, annealed	9158	
Iron, annealed	9827	
Nickel, annealed	12600	
Tin, pressed	13360	·365
Lead, pressed	19850	·387
Antimony, pressed	35900	·389
Bismuth, pressed	132650	·354
Mercury, liquid	96190	·072
Alloy, 2 parts platinum, 1 part silver, by weight, hard or annealed	2466	·031
German silver, hard or annealed	21170	·044
Alloy, 2 parts gold, 1 silver, by weight, hard or annealed	10990	·065

Resistances of Conductors of Telegraphic Cables per nautical mile, at 24° C., in electromagnetic measure.

Red Sea	$7{\cdot}94 \times 10^9$
Malta-Alexandria, mean	3·49 „
Persian Gulf, mean	6·284 „
Second Atlantic, mean	4·272 „

Electromotive Force.

125. The electromotive force of a Daniell's cell was found by Sir W. Thomson (see above, § 121) to be

$$\cdot 00374 \text{ electrostatic unit.}$$

As 1 electrostatic unit is 3×10^{10} electromagnetic units, this is $\cdot 00374 \times 3 \times 10^{10} = 1{\cdot}12 \times 10^8$ electromagnetic units, or 1·12 volt.

According to Latimer Clark's experimental determinations (Journ. Soc. Tel. Eng. January 1873), the electromotive force of

a Daniell's cell is $1\cdot11 \times 10^8$, and the electromotive force of a Grove's cell is $1\cdot97 \times 10^8$.

According to the determination of F. Kohlrausch (Pogg. Ann. vol. cxli. [1870], and Ergänz. vol. vi. [1874] p. 35), the electromotive force of a Daniell's cell is $1\cdot138 \times 10^8$, and that of a Grove's cell $1\cdot942 \times 10^8$.

The electromotive force of Latimer Clark's standard cell (Phil. Mag. June 1872, and Phil. Trans. 1873) is $1\cdot457 \times 10^8$.

For theoretical determinations see § 129.

126. The electromotive force of a thermoelectric circuit is called *Thermoelectric force.*

The following Table, showing the thermoelectric force of a couple of which lead is one element, is from Jenkin's 'Electricity and Magnetism,' p. 176, except that we have employed the multiplier 100 to reduce from microvolts to C.G.S. electromagnetic units. It was compiled from Matthiessen's experiments; and the mean temperature for which it is true may be taken at from 19° to 20° C.

Table of Thermoelectric Forces, in electromagnetic units, for 1° C. of difference of temperature of junctions, lead being one element.

Bismuth, pressed commercial wire	+9700	Antimony, pressed wire	− 280
Bismuth, pure pressed wire	+8900	Silver, pure hard	− 300
Bismuth, crystal, axial	+6500	Zinc, pure pressed	− 370
" equatorial	+4500	Copper, galvanoplastically precipitated	− 380
Cobalt	+2200	Antimony, pressed commercial wire	− 600
German silver	+1175	Arsenic	− 1356
Quicksilver	41·8	Iron, pianoforte wire	− 1750
Lead	0	Antimony, axial	− 2260
Tin	− 10	" equatorial	− 2640
Copper of commerce	− 10	Phosphorus, red	− 2970
Platinum	− 90	Tellurium	−50200
Gold	− 120	Selenium	−80700

127. The following Table is based upon Professor Tait's thermoelectric diagram (Trans. Roy. Soc. Edin. vol. xxvii. [Dec. 1873]), joined with the assumption that a Grove's cell has electromotive force $1\cdot97 \times 10^8$:—

Table of Thermoelectric Values referred to lead as zero.

	Thermoelectric value in electromagnetic units (t denoting temperature Centigrade).
Iron	$-1734+4{\cdot}87\,t$
Steel	$-1139+3{\cdot}28\,t$
Alloy believed to be platinum iridium	-839 at all temperatures.
Alloy, platinum 95; iridium 5	$-622+{\cdot}55\,t$
" " 90 " 10	$-596+1{\cdot}34\,t$
" " 85 " 15	$-709+{\cdot}63\,t$
" " 85 " 15	-577 at all temperatures.
Soft platinum	$+61+1{\cdot}10\,t$
Alloy, platinum and nickel	$-544+1{\cdot}10\,t$
Hard platinum	$-260+{\cdot}75\,t$
Magnesium	$-224+{\cdot}95\,t$
German silver	$+1207+5{\cdot}12\,t$
Cadmium	$-266-4{\cdot}29\,t$
Zinc	$-234-2{\cdot}40\,t$
Silver	$-214-1{\cdot}50\,t$
Gold	$-283-1{\cdot}02\,t$
Copper	$-136-{\cdot}95\,t$
Lead	0
Tin	$+43-{\cdot}55\,t$
Aluminium	$+77-{\cdot}39\,t$
Palladium	$+625+3{\cdot}59\,t$
Nickel to 175° C.	$+2204+5{\cdot}12\,t$
" 250° to 310° C.	$+8449-24{\cdot}1\,t$
" from 340° C.	$+307+5{\cdot}12\,t$

The lower limit of temperature for the Table is −18° C. for all the metals in the list. The upper limit is 416° C., with the following exceptions:—Cadmium, 258° C.; zinc, 373° C.; German silver, 175° C.

The difference of the "thermoelectric values" of two metals for a given temperature t, is the electromotive force per degree of difference between the temperatures of the junctions in a couple formed of these metals, when the mean of the temperatures of the junctions is t. The current through the hot junction is from the metal of higher to that of lower "thermoelectric value."

Example 1.

Required the electromotive force of a copper-iron couple, the temperatures of the junctions being 0° C. and 100° C.

We have, for copper, $\quad -136-{\cdot}95\,t;$

" iron, $\quad -1734+4{\cdot}87\,t;$

" copper−iron, $=1598-5{\cdot}82\,t.$

The electromotive force per degree is $1598-5{\cdot}82\times 50=1307$

electromagnetic units, and the electromotive force of the couple is 1307(100−0) = 130700.

By the *neutral point* of two metals is meant the temperature at which their thermoelectric values are equal.

Example 2.

To find the neutral point of copper and iron we have

$$\text{copper} - \text{iron} = 1598 - 5{\cdot}82\,t = 0,\ t = 275\,;$$

that is, the neutral point is 275° C. When the mean of the temperatures of the junctions is below this point, the current through the warmer junction is from copper to iron. The current ceases as the mean temperature attains the neutral point, and is reversed in passing it.

Example 3.

F. Kohlrausch (Pogg. Ann. Ergänz. vol. vi. p. 35 [1874]) states that, according to his determination, the electromotive force of a couple of iron and German silver is 24×10^5 millimetre-milligramme-second units for 1° of difference of temperatures of the junctions at moderate temperatures. Compare this result with the above Table at mean temperature 100°.

The dimensions of electromotive force are $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$; hence the C.G.S. value of Kohlrausch's unit is $10^{-\frac{3}{2}}10^{-\frac{3}{2}} = 10^{-3}$, giving 2400 as the electromotive force per degree of difference.

From the above Table we have

$$\text{German silver} - \text{iron} = 2941 + {\cdot}25\,t,$$

which, for $t = 100$, gives 2966 as the electromotive force per degree of difference.

Electrochemical Equivalents.

128. The following are examples of electrolytic decompositions which require the same quantity of electricity to effect them:—

Substance decomposed.	Mass decomposed.	Masses of products.			
Water	18	2	hydrogen,	16	oxygen.
Hydrochloric acid	73	2	"	71	chlorine.
Potassium chloride	149	78	potassium,	71	"
Sodium "	117	46	sodium,	71	"
Silver "	287	216	silver,	71	"
Potassium iodide	332	78	potassium,	254	iodine.
" bromide	238	78	"	160	bromine.
Calcium chloride	111	40	calcium,	71	chlorine.
Zinc "	136	65	zinc,	71	"
Ferrous "	127	56	iron,	71	"
Ferric "	$108\frac{1}{3}$	$37\frac{1}{3}$	"	71	"
Cuprous "	198	127	copper,	71	"
Cupric "	$134\frac{1}{2}$	$63\frac{1}{2}$	"	71	"
Mercuric "	271	200	mercury	71	"
Potassium sulphate	174	78	potassium.		
Zinc "	163	65	zinc.		
Lead nitrate	331	207	lead.		
Silver "	340	216	silver.		
Stannous chloride	189	118	tin,	71	chlorine.
Stannic "	130	59	"	71	"

129. According to the experiments of F. Kohlrausch (Pogg. Ann. vol. cxlix [1873]), the quantity of silver deposited by the C.G.S. unit (electromagnetic) of electricity is ·011363 gramme. Hence $\frac{216}{\cdot 011363}$ or 19010 is the quantity of electricity required to produce the above effects if the numbers are taken as denoting grammes.

Let W ergs be the chemical work done in a cell of a battery for 65 grammes of zinc consumed (being the heat of chemical combination multiplied by Joule's equivalent); then $\frac{W}{19010}$ will be the electromotive force of the cell, on the supposition that there is no wasteful action.

According to a calculation by Professor G. C. Foster, based on Julius Thomsen's determinations of the heat of combination, 65 grammes of zinc consumed correspond to

40105	gramme-degrees in	Smee's cell.
52347	"	Daniell's cell.
90162	"	Grove's cell.

Multiplying by $\frac{4{\cdot}2 \times 10^7}{19010}$ we obtain

$8{\cdot}86 \times 10^7$	as the electromotive	force of	Smee's cell.
$1{\cdot}156 \times 10^8$	"	"	Daniell's cell.
$1{\cdot}991 \times 10^8$	"	"	Grove's cell.

These results are slightly in excess of the values obtained by direct observation (see § 125).

130. *Examples in Electricity.*

1. Two conducting spheres, each of 1 centim. radius, are placed at a distance of r centims. from centre to centre, r being a large number; and each of them is charged with an electrostatic unit of positive electricity. With what force will they repel each other?

Since r is large, the charge may be assumed to be uniformly distributed over their surfaces, and the force will be the same as if the charge of each were collected at its centre. The force will therefore be $\frac{1}{r^2}$ of a dyne.

2. Two conducting spheres, each of 1 centim. radius, placed as in the preceding question, are connected one with each pole of a Daniell's battery by means of two very fine wires whose capacity may be neglected, so that the capacity of each sphere when thus connected is sensibly equal to unity. Of how many cells must the battery consist that the spheres may attract each other with a force of $\frac{1}{r^2}$ of a dyne, r being the distance between their centres in centims.?

One sphere must be charged to potential 1 and the other to potential -1; or, more generally, the difference of their potentials must be 2. The number of cells required is

$$\frac{2}{\cdot 000374} = 5350.$$

3. How many Daniell's cells would be required to produce a spark between two parallel conducting surfaces at a distance of $\cdot 019$ of a centim., and how many at a distance of $\cdot 0086$ of a centim.? (See §§ 120, 121.)

Ans. $\frac{4 \cdot 26}{\cdot 000374} = 11390$; $\frac{2 \cdot 30}{\cdot 000374} = 6150$.

4. Compare the capacity denoted by 1 farad with the capacity of the earth.

The capacity of the earth in static measure is equal to its radius, namely $6{\cdot}37 \times 10^8$. Dividing by v^2 to reduce to magnetic measure, we have $\cdot 71 \times 10^{-12}$, which is 1 farad multiplied by $\cdot 71 \times 10^{-3}$, or is $\cdot 00071$ of a farad. A farad is therefore 1400 times the capacity of the earth.

5. Calculate the resistance of a cell consisting of a plate of zinc, A square centims. in area, and a plate of copper of the same dimensions separated by an acid solution of specific resistance 10^9, the distance between the plates being 1 centim.

Ans. $\frac{10^9}{A}$, or $\frac{1}{A}$ of an ohm.

6. Find the heat developed in 10 minutes by the passage of a current from 10 Daniell's cells in series through a wire of resistance 10^{10} (that is, 10 ohms), assuming the electromotive force of each cell to be $1{\cdot}1 \times 10^8$, and the resistance of each cell to be 10^9.

Here we have

$$\begin{aligned}
&\text{Total electromotive force} &&= 1{\cdot}1 \times 10^9. \\
&\text{Resistance in battery} &&= 10^{10}. \\
&\text{Resistance in wire} &&= 10^{10}. \\
&\text{Current} = \frac{1{\cdot}1 \times 10^9}{2 \times 10^{10}} &&= \cdot 55 \times 10^{-1} = \cdot 055. \\
&\left.\begin{array}{l}\text{Heat developed in} \\ \quad\text{wire per second}\end{array}\right\} &&= \frac{(\cdot 055)^2 \times 10^{10}}{4{\cdot}2 \times 10^7} = \cdot 7204.
\end{aligned}$$

Hence the heat developed in 10 minutes is $432{\cdot}24$ gramme-degrees.

7. Find the electromotive force between the wheels on opposite sides of a railway carriage travelling at the rate of 30 miles an hour on a narrow-gauge line [4 feet $8\frac{1}{2}$ inches] due to cutting the lines of force of terrestrial magnetism, the vertical intensity being $\cdot 438$.

The electromotive force will be the product of the velocity of travelling, the distance between the rails, and the vertical intensity; that is,

$$(44{\cdot}7 \times 30)(2{\cdot}54 \times 56{\cdot}5)(\cdot 438) = 84300 \text{ electromagnetic units.}$$

This is about $\frac{1}{1200}$ of a volt.

8. Find the electromotive force at the instant of passing the magnetic meridian, in a circular coil consisting of 300 turns of wire, revolving at the rate of 10 revolutions per second about a vertical diameter; the diameter of the coil being 30 centims., and the horizontal intensity of terrestrial magnetism being ·1794, no other magnetic influence being supposed present.

The numerical value of the lines of force which go through the coil when inclined at an angle θ to the meridian, is the horizontal intensity multiplied by the area of the coil and by $\sin\theta$; say $nH\pi a^2 \sin\theta$, where $H = {\cdot}1794$, $a = 15$, and $n = 300$. The electromotive force at any instant is the rate at which this quantity increases or diminishes; that is, $nH\pi a^2 \cos\theta \,.\, \omega$, if ω denote the angular velocity. At the instant of passing the meridian $\cos\theta$ is 1, and the electromotive force is $nH\pi a^2\omega$. With 10 revolutions per second the value of ω is $2\pi \times 10$.

Hence the electromotive force is

$$\cdot 1794 \times (3{\cdot}142)^2 \times 225 \times 20 \times 300 = 2{\cdot}39 \times 10^6.$$

This is about $\frac{1}{42}$ of a volt.

131. To investigate the magnitudes of units of length, mass, and time which will fulfil the three following conditions:—

1. The acceleration due to the attraction of unit mass at unit distance shall be unity.

2. The electrostatic units shall be equal to the electromagnetic units.

3. The density of water at 4° C. shall be unity.

Let the 3 units required be equal respectively to L centims., M grammes, and T seconds.

We have in C.G.S. measure, for the acceleration due to attraction (§ 53),

$$\text{acceleration} = C \frac{\text{mass}}{(\text{distance})^2}, \text{ where } C = 6{\cdot}48 \times 10^{-8};$$

and in the new system we are to have

$$\text{acceleration} = \frac{\text{mass}}{(\text{distance})^2}.$$

Hence, by division,

$$\frac{\text{acceleration in C.G.S. units}}{\text{acceleration in new units}}$$

$$= \mathrm{C}\,\frac{\text{mass in C.G.S. units}}{\text{mass in new units}}\cdot\frac{(\text{distance in new units})^2}{(\text{distance in C.G.S. units})^2};$$

that is, $\frac{\mathrm{L}}{\mathrm{T}^2}=\mathrm{C}\frac{\mathrm{M}}{\mathrm{L}^2}$.

This equation expresses the first of the three conditions.

The equation $\frac{\mathrm{L}}{\mathrm{T}}=v$ expresses the second, v denoting 3×10^{10}.

The equation $\mathrm{M}=\mathrm{L}^3$ expresses the third.

Substituting L^3 for M in the first equation, we find $\mathrm{T}=\sqrt{\frac{1}{\mathrm{C}}}$.

Hence, from the second equation,

$$\mathrm{L}=v\sqrt{\frac{1}{\mathrm{C}}};$$

and from the third,

$$\mathrm{M}=\left(v\sqrt{\frac{1}{\mathrm{C}}}\right)^3.$$

Introducing the actual values of C and v, we have approximately

$$\mathrm{T}=3928,\quad \mathrm{L}=1{\cdot}178\times10^{14},\quad \mathrm{M}=1{\cdot}63\times10^{42};$$

that is to say, the new unit of time will be about $1^{\mathrm{h}}\ 5\frac{1}{2}^{\mathrm{m}}$;

the new unit of length will be about 118 thousand earth-quadrants;

the new unit of mass will be about $2{\cdot}66\times10^{14}$ times the earth's mass.

Electrodynamics.

132. Ampère's formula for the repulsion between two elements of currents, when expressed in electromagnetic units, is

$$\frac{cc'\,ds\,.\,ds'}{r^2}(2\sin\alpha\sin\alpha'\cos\theta-\cos\alpha\cos\alpha'),$$

where c, c' denote the strengths of the two currents;

ds, ds' the lengths of the two elements;

α, α' the angles which the elements make with the line joining them;

r the length of this joining line;

θ the angle between the plane of r, ds, and the plane of r, ds'.

For two parallel currents, one of which is of infinite length, and the other of length l, the formula gives by integration an attraction or repulsion,

$$\frac{2l}{\mathrm{D}}cc',$$

where D denotes the perpendicular distance between the currents.

Example.

Find the attraction between two parallel wires a metre long and a centim. apart when a current of $\frac{1}{10}$ is passing through each. Here the attraction will be sensibly the same as if one of the wires were indefinitely increased in length, and will be

$$\frac{200}{1}\left(\frac{1}{10}\right)^2=2;$$

that is, each wire will be attracted or repelled with a force of 2 dynes, according as the directions of the currents are the same or opposite.

APPENDIX.

First Report of the Committee for the Selection and Nomenclature of Dynamical and Electrical Units, the Committee consisting of Sir W. Thomson, F.R.S., Professor G. C. Foster, F.R.S., Professor J. C. Maxwell, F.R.S., Mr. G. J. Stoney, F.R.S.*, Professor Fleeming Jenkin, F.R.S., Dr. Siemens, F.R.S., Mr. F. J. Bramwell, F.R.S., *and* Professor Everett (Reporter).

We consider that the most urgent portion of the task intrusted to us is that which concerns the selection and nomenclature of units of force and energy; and under this head we are prepared to offer a definite recommendation.

A more extensive and difficult part of our duty is the selection and nomenclature of electrical and magnetic units. Under this head we are prepared with a definite recommendation as regards selection, but with only an interim recommendation as regards nomenclature.

Up to the present time it has been necessary for every person who wishes to specify a magnitude in what is called "absolute" measure, to mention the three fundamental units of mass, length, and time which he has chosen as the basis of his system. This necessity will be obviated if one definite selection of three fundamental units be made once for all, and accepted by the general consent of scientific men. We are strongly of opinion that such a selection ought at once to be made, and to be so made that there will be no subsequent necessity for amending it.

We think that, in the selection of each kind of derived unit,

* Mr. Stoney did not concur in the recommendations of this Report, and is not responsible for the C.G.S. system.

all arbitrary multiplications and divisions by powers of ten, or other factors, must be rigorously avoided, and the whole system of fundamental units of force, work, electrostatic, and electro-magnetic elements must be fixed at one common level—that level, namely, which is determined by direct derivation from the three fundamental units once for all selected.

The carrying out of this resolution involves the adoption of some units which are excessively large or excessively small in comparison with the magnitudes which occur in practice; but a remedy for this inconvenience is provided by a method of denoting decimal multiples and submultiples, which has already been extensively adopted, and which we desire to recommend for general use.

On the initial question of the particular units of mass, length, and time to be recommended as the basis of the whole system, a protracted discussion has been carried on, the principal point discussed being the claims of the gramme, the *metre,* and the second, as against the gramme, the *centimetre,* and the second,—the former combination having an advantage as regards the simplicity of the name *metre,* while the latter combination has the advantage of making the unit of mass practically identical with the mass of unit-volume of water—in other words, of making the value of the density of water practically equal to unity. We are now all but unanimous in regarding this latter element of simplicity as the more important of the two; and in support of this view we desire to quote the authority of Sir W. Thomson, who has for a long time insisted very strongly upon the necessity of employing units which conform to this condition.

We accordingly recommend the general adoption of the *Centimetre,* the *Gramme,* and the *Second* as the three fundamental units; and until such time as special names shall be appropriated to the units of electrical and magnetic magnitude hence derived, we recommend that they be distinguished from "absolute" units otherwise derived, by the letters "C.G.S." prefixed, these being the initial letters of the names of the three fundamental units.

Special names, if short and suitable, would, in the opinion of a majority of us, be better than the provisional designations

"C.G.S. unit of . . . ." Several lists of names have already been suggested; and attentive consideration will be given to any further suggestions which we may receive from persons interested in electrical nomenclature.

The "ohm," as represented by the original standard coil, is approximately 10^9 C.G.S. units of resistance; the "volt" is approximately 10^8 C.G.S. units of electromotive force; and the "farad" is approximately $\frac{1}{10^9}$ of the C.G.S. unit of capacity.

For the expression of high decimal multiples and submultiples, we recommend the system introduced by Mr. Stoney, a system which has already been extensively employed for electrical purposes. It consists in denoting the exponent of the power of 10, which serves as multiplier, by an appended cardinal number, if the exponent be positive, and by a prefixed ordinal number if the exponent be negative.

Thus 10^9 grammes constitute a *gramme-nine*; $\frac{1}{10^9}$ of a gramme constitutes a *ninth-gramme*; the approximate length of a quadrant of one of the earth's meridians is a *metre-seven*, or a *centimetre-nine*.

For multiplication or division by a million, the prefixes *mega** and *micro* may conveniently be employed, according to the present custom of electricians. Thus the *megohm* is a million ohms, and the *microfarad* is the millionth part of a farad. The prefix *mega* is equivalent to the affix *six*. The prefix *micro* is equivalent to the prefix *sixth*.

The prefixes *kilo, hecto, deca, deci, centi, milli* can also be employed in their usual senses before all new names of units.

As regards the name to be given to the C.G.S. *unit of force*, we recommend that it be a derivative of the Greek δύναμις. The form *dynamy* appears to be the most satisfactory to etymologists. *Dynam* is equally intelligible, but awkward in sound to English ears. The shorter form, *dyne*, though not fashioned according to strict rules of etymology, will probably be generally preferred in this country. Bearing in mind that it is desirable

* Before a vowel, either *meg* or *megal*, as euphony may suggest, may be employed instead of *mega*.

to construct a system with a view to its becoming international, we think that the termination of the word should for the present be left an open question. But we would earnestly request that, whichever form of the word be employed, its meaning be strictly limited to the unit of force of the C.G.S. system—that is to say, *the force which, acting upon a gramme of matter for a second, generates a velocity of a centimetre per second.*

The C.G.S. *unit of work* is the work done by *this force working through a centimetre*; and we propose to denote it by some derivative of the Greek ἔργον. The forms *ergon*, *ergal*, and *erg* have been suggested; but the second of these has been used in a different sense by Clausius. In this case also we propose, for the present, to leave the termination unsettled; and we request that the word *ergon*, or *erg*, be strictly limited to the C.G.S. unit of work, or what is, for purposes of measurement, equivalent to this, the C.G.S. *unit of energy*, energy being measured by the amount of work which it represents.

The C.G.S. *unit of power* is the power of doing work at the rate of *one erg per second*; and the power of an engine, under given conditions of working, can be specified in *ergs per second.*

For rough comparison with the vulgar (and variable) units based on terrestrial gravitation, the following statement will be useful:—

The *weight* of a *gramme*, at any part of the earth's surface, is about 980 *dynes*, or rather less than a *kilodyne.*

The *weight* of a *kilogramme* is rather less than a *megadyne*, being about 980,000 *dynes.*

Conversely, the *dyne* is about 1·02 times the *weight* of a *milligramme* at any part of the earth's surface; and the *megadyne* is about 1·02 times the *weight* of a *kilogramme.*

The *kilogrammetre* is rather less than the *ergon-eight*, being about 98 million *ergs.*

The *gramme-centimetre* is rather less than the *kilerg*, being about 980 *ergs.*

For exact comparison, the value of g (the acceleration of a body falling in vacuo) at the station considered must of course be known. In the above comparisons it is taken as 980 C.G.S. units of acceleration.

One *horse-power* is about three quarters of an *erg-ten* per second. More nearly, it is 7·46 *erg-nines* per second, and one *force-de-cheval* is 7·36 *erg-nines* per second.

The mechanical equivalent of one *gramme-degree* (Centigrade) of heat is 41·6 megalergs, or 41,600,000 *ergs*.

Second Report of the Committee for the Selection and Nomenclature of Dynamical and Electrical Units, the Committee consisting of Professor Sir W. THOMSON, F.R.S., Professor G. C. FOSTER, F.R.S., Professor J. CLERK MAXWELL, F.R.S., G. J. STONEY, F.R.S., Professor FLEEMING JENKIN, F.R.S., Dr. C. W. SIEMENS, F.R.S., F. J. BRAMWELL, F.R.S., Professor W. G. ADAMS, F.R.S., Professor BALFOUR STEWART, F.R.S., *and* Professor EVERETT (Secretary).

THE Committee on the Nomenclature of Dynamical and Electrical Units have circulated numerous copies of their last year's Report among scientific men both at home and abroad.

They believe, however, that, in order to render their recommendations fully available for science teaching and scientific work, a full and popular exposition of the whole subject of physical units is necessary, together with a collection of examples (tabular and otherwise) illustrating the application of systematic units to a variety of physical measurements. Students usually find peculiar difficulty in questions relating to units; and even the experienced scientific calculator is glad to have before him concrete examples with which to compare his own results, as a security against misapprehension or mistake.

Some members of the Committee have been preparing a small volume of illustrations of the C.G.S. system [Centimetre-Gramme-Second system] intended to meet this want.

[The present work is the volume of illustrations here referred to].

ERRATUM.

Page 46, line 2, for *centimetres* read *centimetre.*

INDEX.

The numerals refer to the pages.